

Lecture 9

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
 - Tuesday 16-18, ~~Online~~, P8
 - E – 50% final grade
 - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
 - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
 - 3att.=+0.5p
 - **all materials/equipments authorized**

2023/2024

- Laboratory – **associate professor Radu Damian**
 - Tuesday 08-12, 11.13 / (08:10)
 - L – 25% final grade
 - ADS, 4 sessions
 - Attendance + **personal results**
 - P – 25% final grade
 - ADS, 3 sessions (-1? 20.02.2024)
 - personal homework

Materials

■ <http://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser window with the URL http://rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0. The page features a dark blue navigation bar with links for Main, Courses, Master, Staff, Research, Students, and Admin. Below this is a secondary navigation bar with links for Microwave CD, Optical Communications, Optoelectronics, Internet, Antennas, Practica, Networks, and Educational software. The main content area is titled "Microwave Devices and Circuits for Radiocommunications (English)" and includes the following information:

- Course: MDCR (2017-2018)**
- Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian
- Code:** EDOS412T
- Discipline Type:** DOS; Alternative, Specialty
- Credits:** 4
- Enrollment Year:** 4, Sem. 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: Examen

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Attendance

[Course](#)
[Laboratory](#)

Lists

[Bonus-uri acumulate \(final\)](#)
[Studenti care nu pot intra in examen](#)

Materials

Course Slides

- [MDCR Lecture 1](#) (pdf, 5.43 MB, en, [↗](#))
- [MDCR Lecture 2](#) (pdf, 3.67 MB, en, [↗](#))
- [MDCR Lecture 3](#) (pdf, 4.76 MB, en, [↗](#))
- [MDCR Lecture 4](#) (pdf, 5.58 MB, en, [↗](#))

On the right side of the screenshot, there is a banner for "RF-OPTO" with the ETTI logo and the University of Technical Sciences (UTS) logo. The banner includes a language selection menu with "English" (circled in red) and "Romana". Below the banner is another navigation bar with links for Main, Courses, Master, Staff, and Research. A secondary navigation bar below that has links for Grades, Student List, Exams, and Photos. The main content area on the right is titled "Online Exams" and includes the text: "In order to participate at online exams you must get ready following..."

Site



English | Romana

[Main](#) [Courses](#) [Master](#) [Staff](#) [Research](#) [Students](#)

Microwave and Optoelectronics Laboratory



We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (ETTI) from the "Gh. Asachi" Technical University (TUJIASI) in Iasi, Romania

We currently cover inside ETTI the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	DCMR	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	details
2	Monolithic Microwave Integrated Circuits	CIMM	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	details
3	Advanced Techniques in the Design of the Radio-communications Systems	TAPSR	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	details
4	Optical Communications	CO	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquium	details
5	Optical Communications	OC	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	details
6	Satellite Communications	CS	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	details
7	Applied Informatics 1	IA1	DOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
8	Applied Informatics 1	AI1	EDOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
9	Databases, Web Programming and Interfacing	DWPI	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verification	details
10	Web Applications Design	PAW	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	details
11	Optoelectronics	OPTO	DID405M	DID	8	4	0P,1L,0S,2C	Colloquium	details
12	Microwave Devices and Circuits for Radiocommunications (English)	MDCR	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	details



Materials

- RF-OPTO
 - <http://rf-opto.etti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**
Wiley; 4th edition , 2011
 - 1 exam problem ← Pozar
- Photos
 - sent by email/**online exam > Week4-Week6**
 - used at lectures/laboratory

Online – Registration no.

- access to **online exams** requires the **password** received by email

The password is communicated during the lectures. It is necessary to

Password

Registration no.

Name of the student

Proposed email 1

Proposed email 2

Write the code below

 **RF-OPTO** 

English | Romana |

[Main](#) [Courses](#) [Master](#) [Staff](#) [Research](#) [Students](#)

[Login](#) [Tutoring](#)

Login

Use the Registration no. and your email or the password received by email

Registration no.

Email/Password

Write the code below

Password

■ received by email

Important message from RF-OPTO Inbox x

 **Radu-Florin Damian**
to me, POPESCU ▾

🗣️ Romanian ▾ > English ▾ [Translate message](#)



Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: ██████████

Identificati-va pe [server](#), cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: ██████████

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

[↩ Reply](#) [↩ Reply all](#) [➡ Forward](#)

Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MD/CR exam from 02/05/2020	██████████
██████████	██████████

From: Me <rdamian@etti.tuiasi.ro> ★
Subject: **Important message from RF-OPTO**
To: ██████████
Cc: Me <rdamian@etti.tuiasi.ro> ★



Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
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In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: ██████████

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Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: ██████████

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Online exam manual

- The online exam app used for:
 - ~~lectures (attendance)~~
 - laboratory
 - project
 - ~~examinations~~

Materials

Other data

[Manual examen on-line](#) (pdf, 2.65 MB, ro, 🇷🇴)

[Simulare Examen](#) (video) (mp4, 65.12 MB, ro, 🇷🇴)

Microwave Devices and Circuits (Englis

Examen online

- always against a **timetable**
 - long period (lecture attendance/laboratory results)
 - ~~short period (tests: 15min, exam: 2h)~~

Announcement 23:59 (10/05/2020)	Support material 00:05 (11/05/2020)	Exam Topics 00:07 (11/05/2020)	Results 00:10 (11/05/2020)	End 00:20 (15/05/2020)	Confirmation 00:20 (16/05/2020)	Next timeframe in: 05 m 43 s Refresh now
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Announcement

This is a "fake" exam, introduced to familiarize you with the server interface and to perform the necessary actions during an exam: thesis scan, selfie, use email for co

Server Time

All exams are based on the server's time zone (it may be different from local time). For reference time on the server is now:

10/05/2020 23:59:16

Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parametri S	T2, fisier parametri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6	Zo6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8	31.6
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0	31.6
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7	31.6
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5	31.6
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2	31.6
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9	31.6
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1	31.6

Online results submission

- many numerical values

	Z1	Z2	Z3	Z4	Z5	Z6	Z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10.692
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



Online results submission

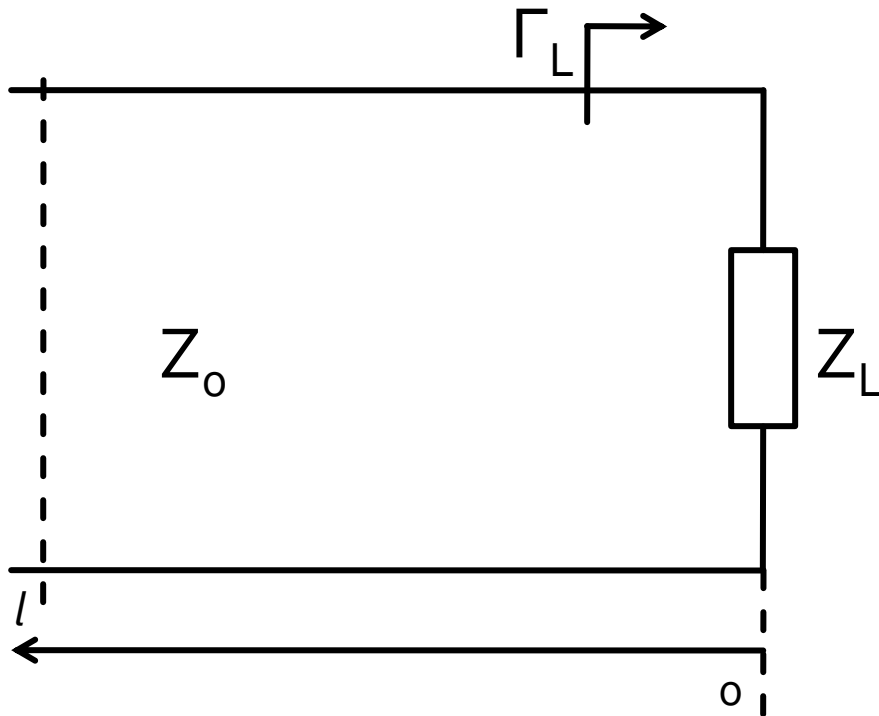
**Grade = Quality of the work +
+ Quality of the submission**

TEM transmission lines

Course Topics

- **Transmission lines**
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

The lossless line



$$V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_0 real

The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta \cdot z} + \Gamma \cdot e^{j\beta \cdot z})$$

$$I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta \cdot z} - \Gamma \cdot e^{j\beta \cdot z})$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\{1 - \Gamma^* \cdot e^{-2j\beta \cdot z} + \Gamma \cdot e^{2j\beta \cdot z} - |\Gamma|^2\}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

$$(z - z^*) = \text{Im}$$

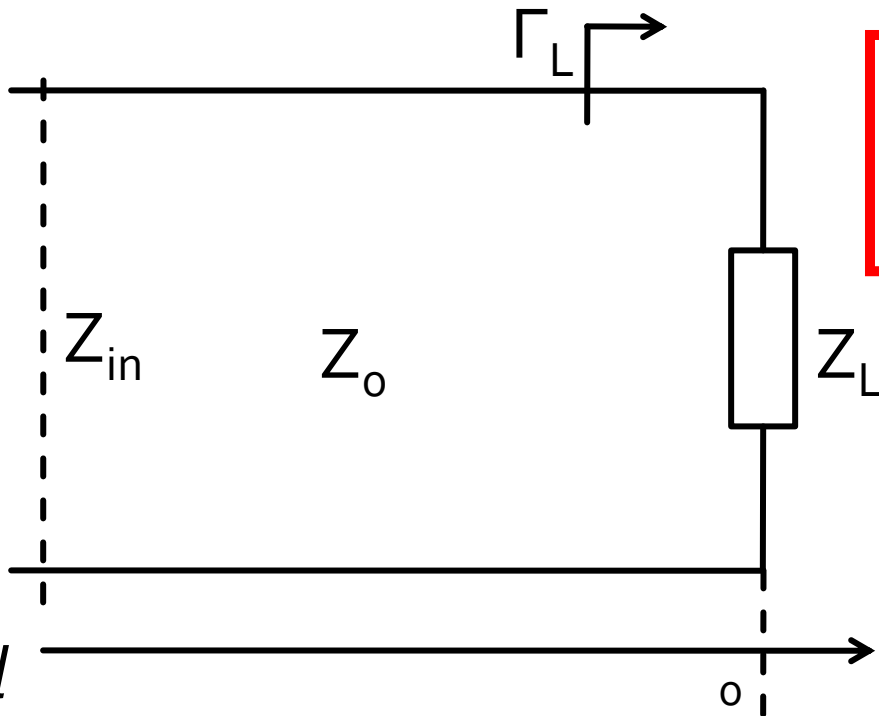
- Total power delivered to the load = Incident power – “Reflected” power

- Return “Loss” [dB]

$$RL = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

The lossless line

- input impedance of a length l of transmission line with characteristic impedance Z_0 , loaded with an arbitrary impedance Z_L

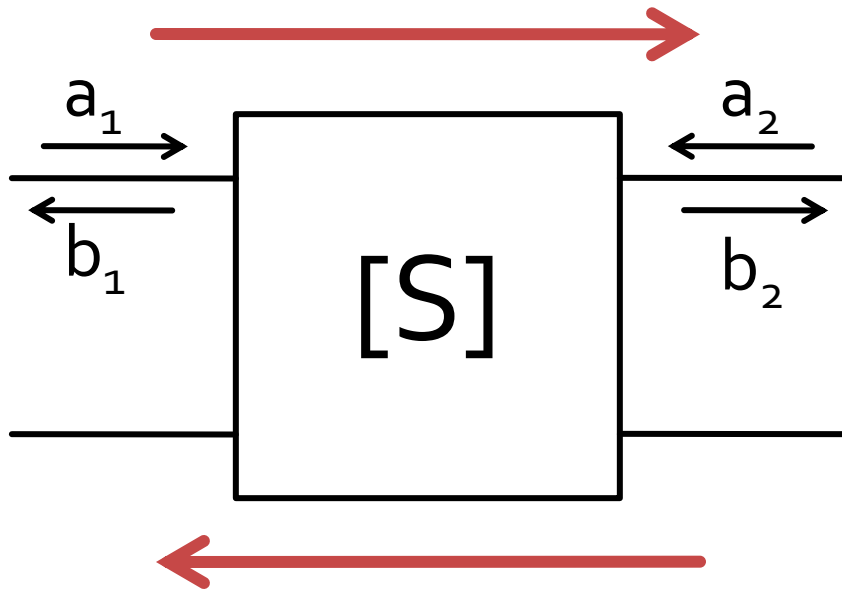


$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

General theory

Microwave Network Analysis

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

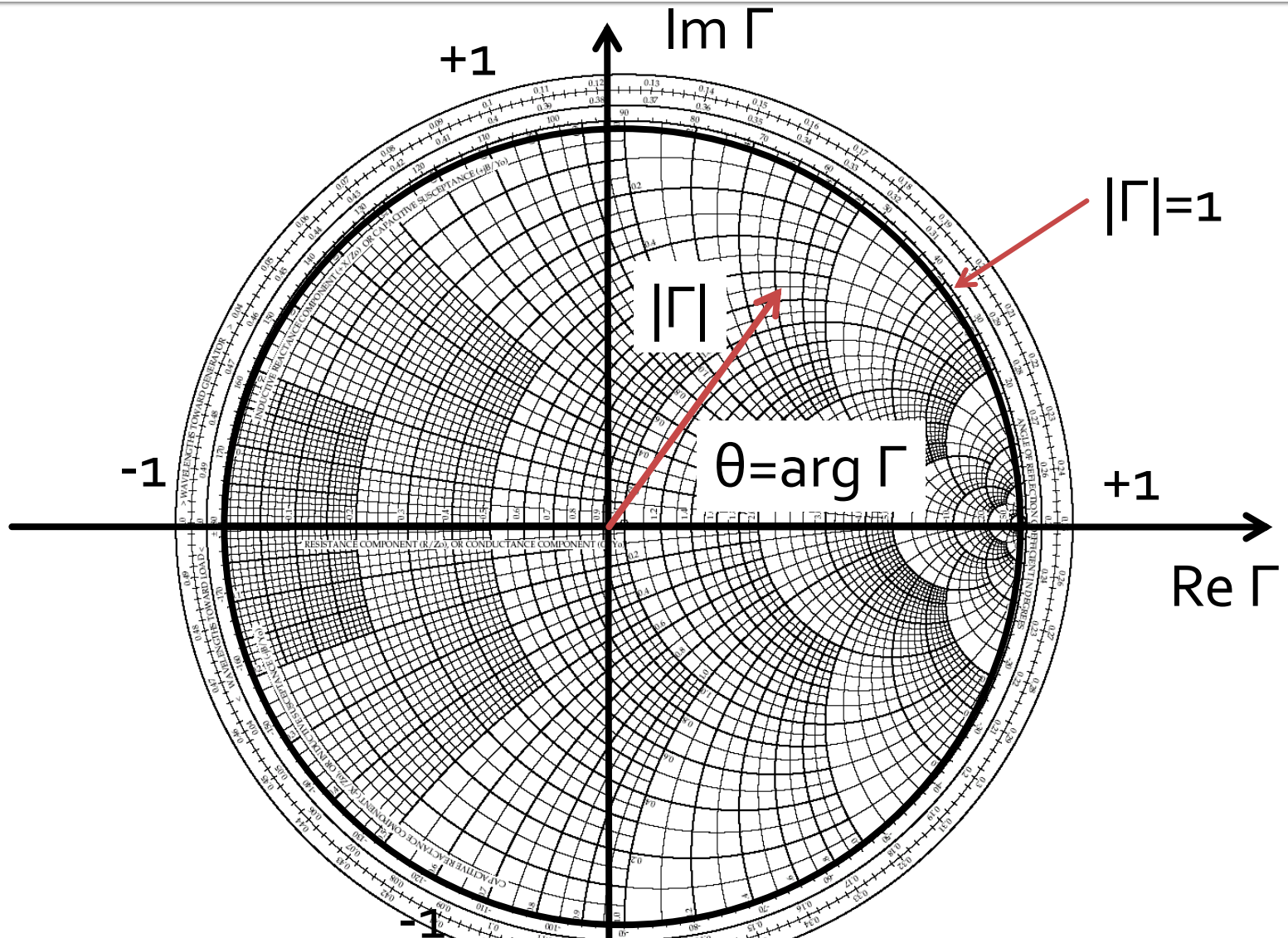
$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- a, b
 - information about signal power **AND** signal phase
- S_{ij}
 - network effect (gain) over signal power **including** phase information

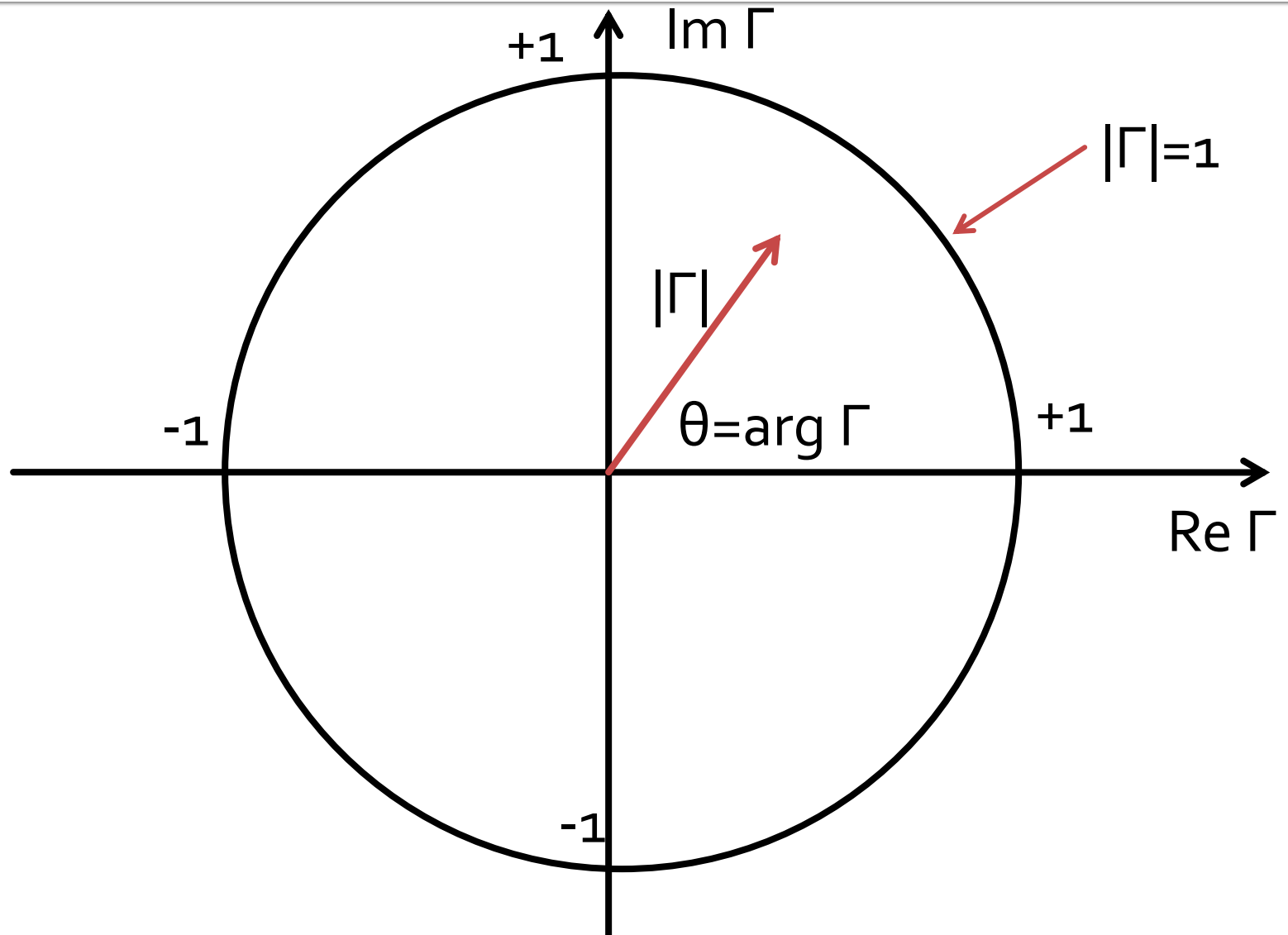
Impedance Matching

The Smith Chart

The Smith Chart



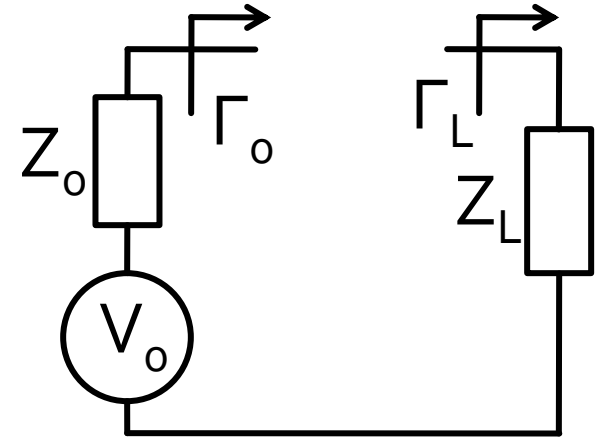
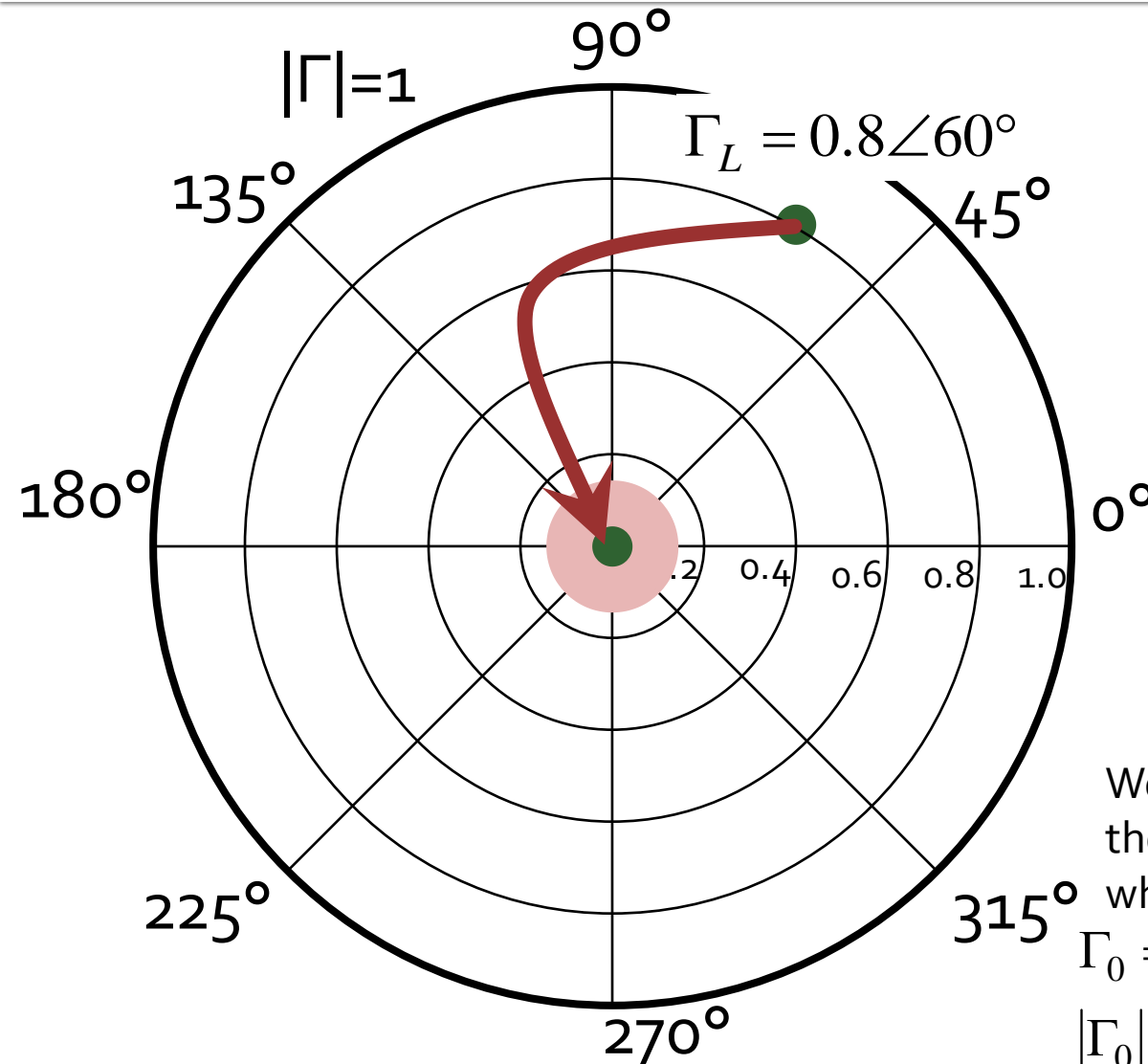
The Smith Chart



Impedance matching

Impedance Matching with lumped elements (L Networks)

The Smith Chart, reflection coefficient, impedance matching



Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

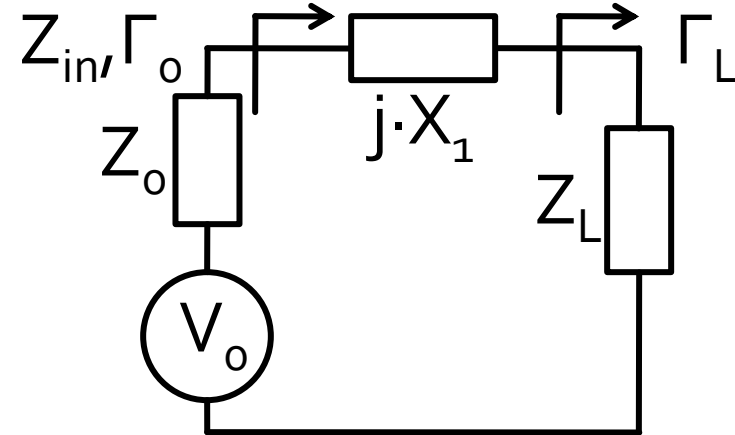
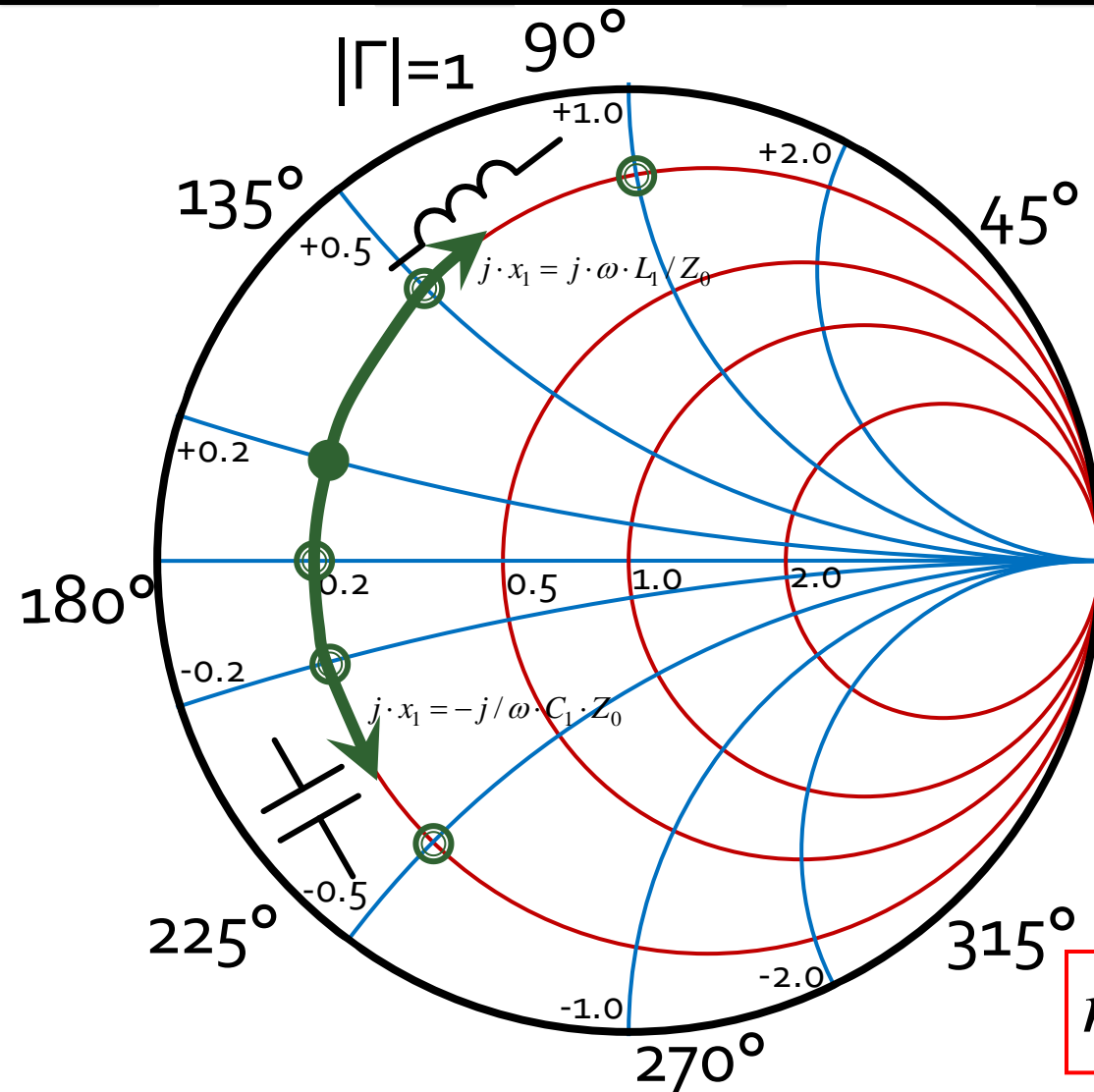
$$\Gamma_L = 0.8 \angle 60^\circ$$

We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$$\Gamma_0 = 0 \text{ perfect match } \bullet$$

$$|\Gamma_0| \leq \Gamma_m \text{ "good enough" match } \bullet$$

The Smith Chart, series reactance



$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678 \angle 156.5^\circ$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1)$$

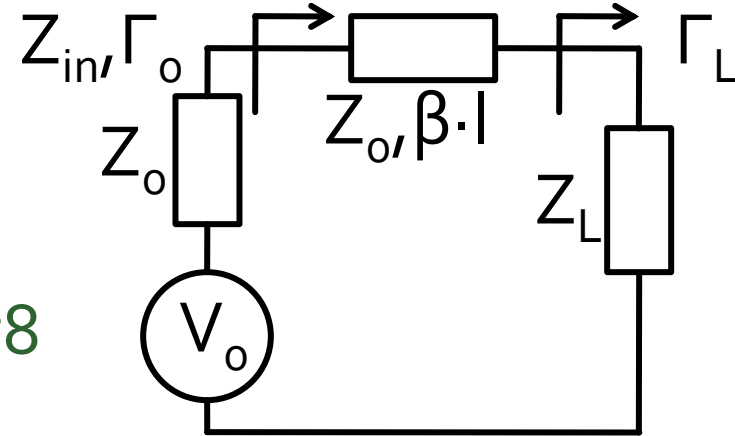
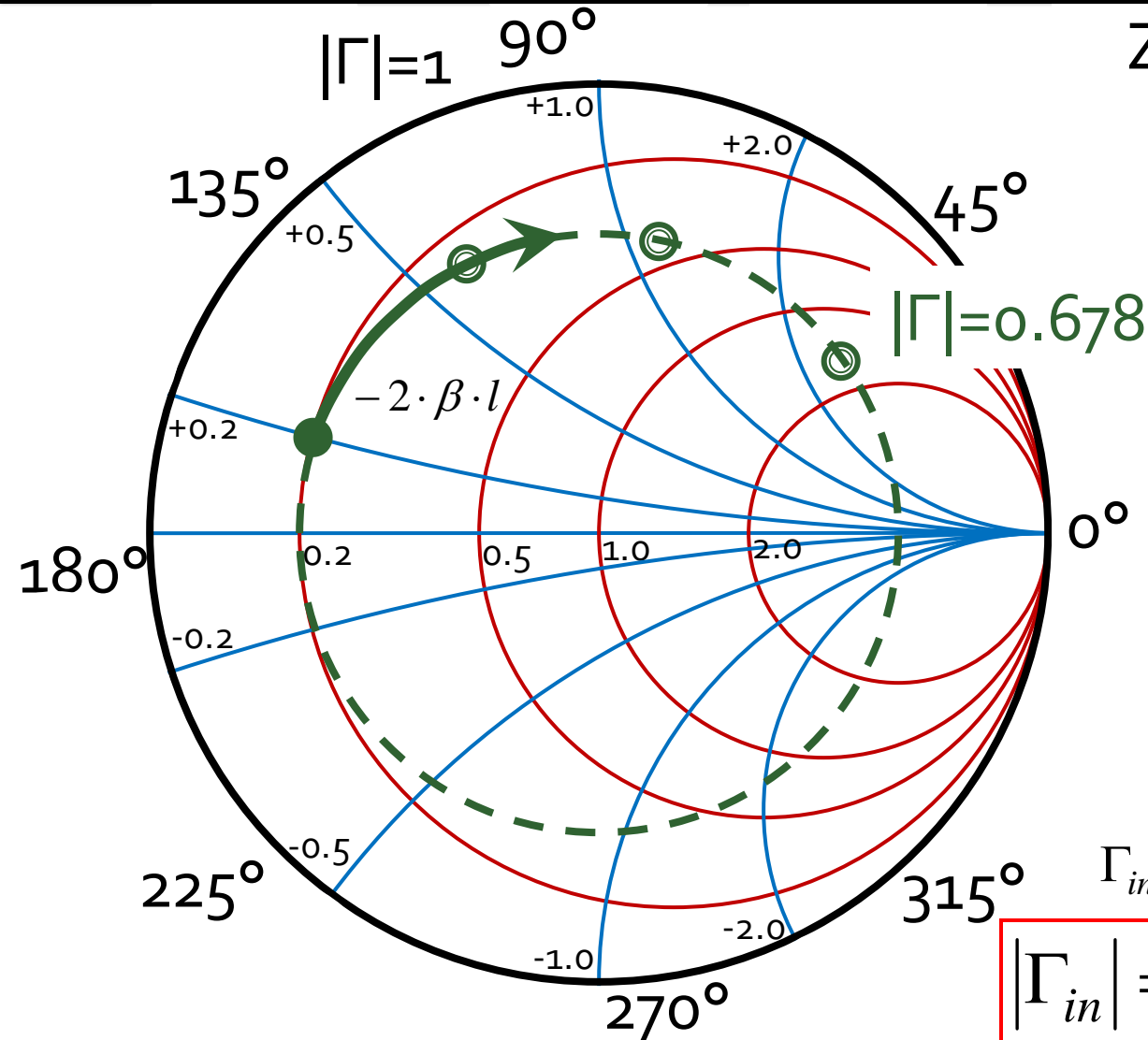
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

The Smith Chart, series transmission line, Z_0



$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

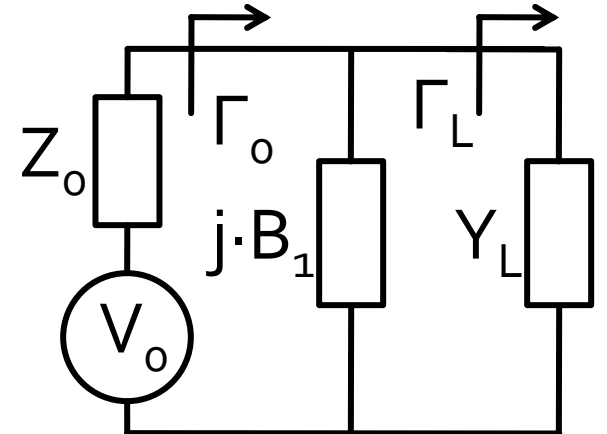
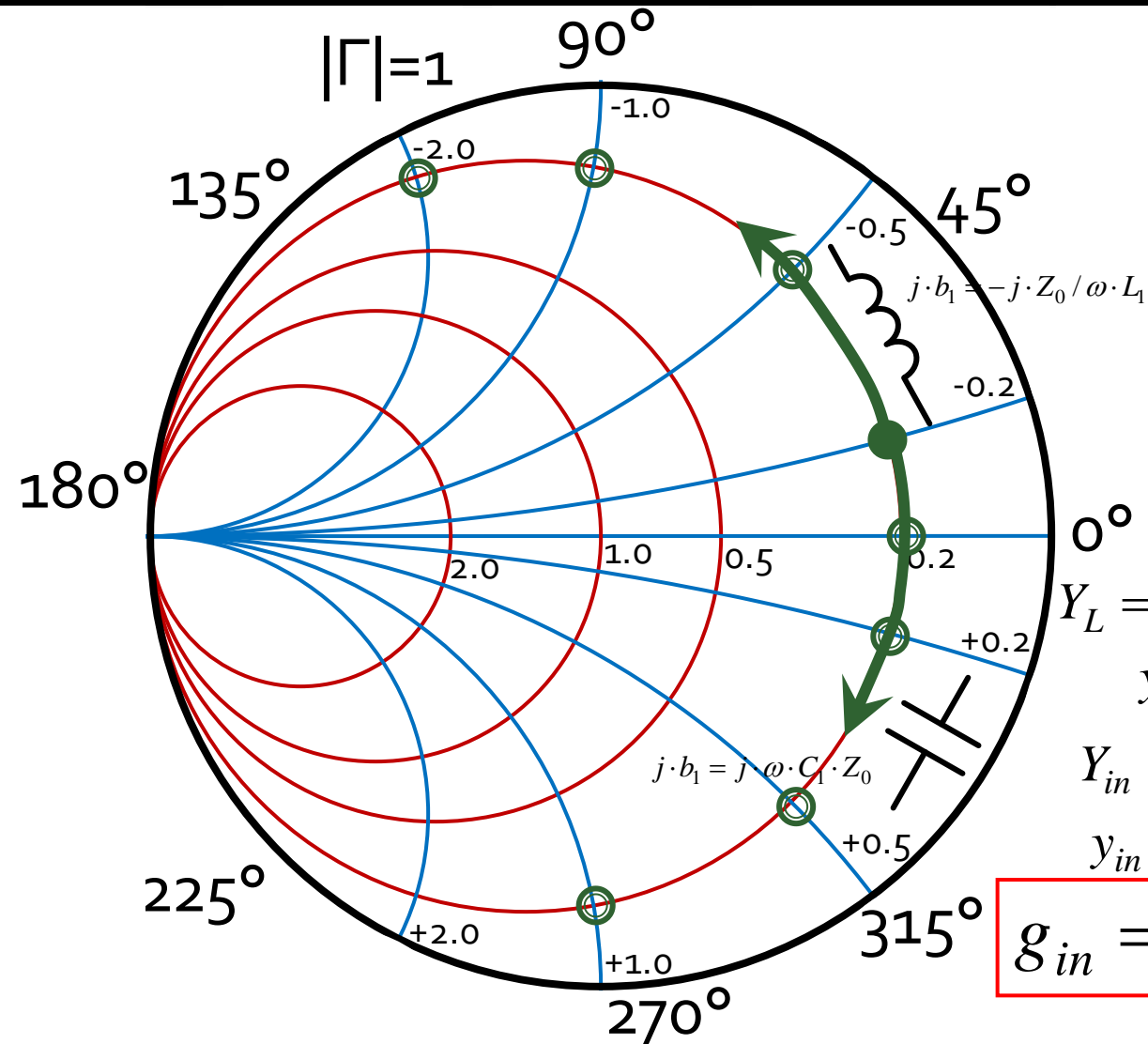
$$\Gamma_L = 0.678 \angle 156.5^\circ$$

$$Z_{in} = Z_0 \cdot \frac{1 + \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}}{1 - \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}$$

$$|\Gamma_{in}| = |\Gamma_L| \quad \arg(\Gamma_{in}) = \arg(\Gamma_L) - 2 \cdot \beta \cdot l$$

The Smith Chart, shunt susceptance



$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678 \angle 23.5^\circ$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

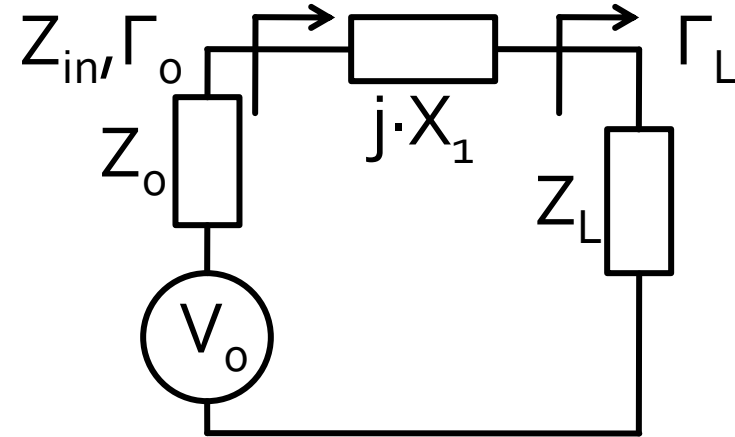
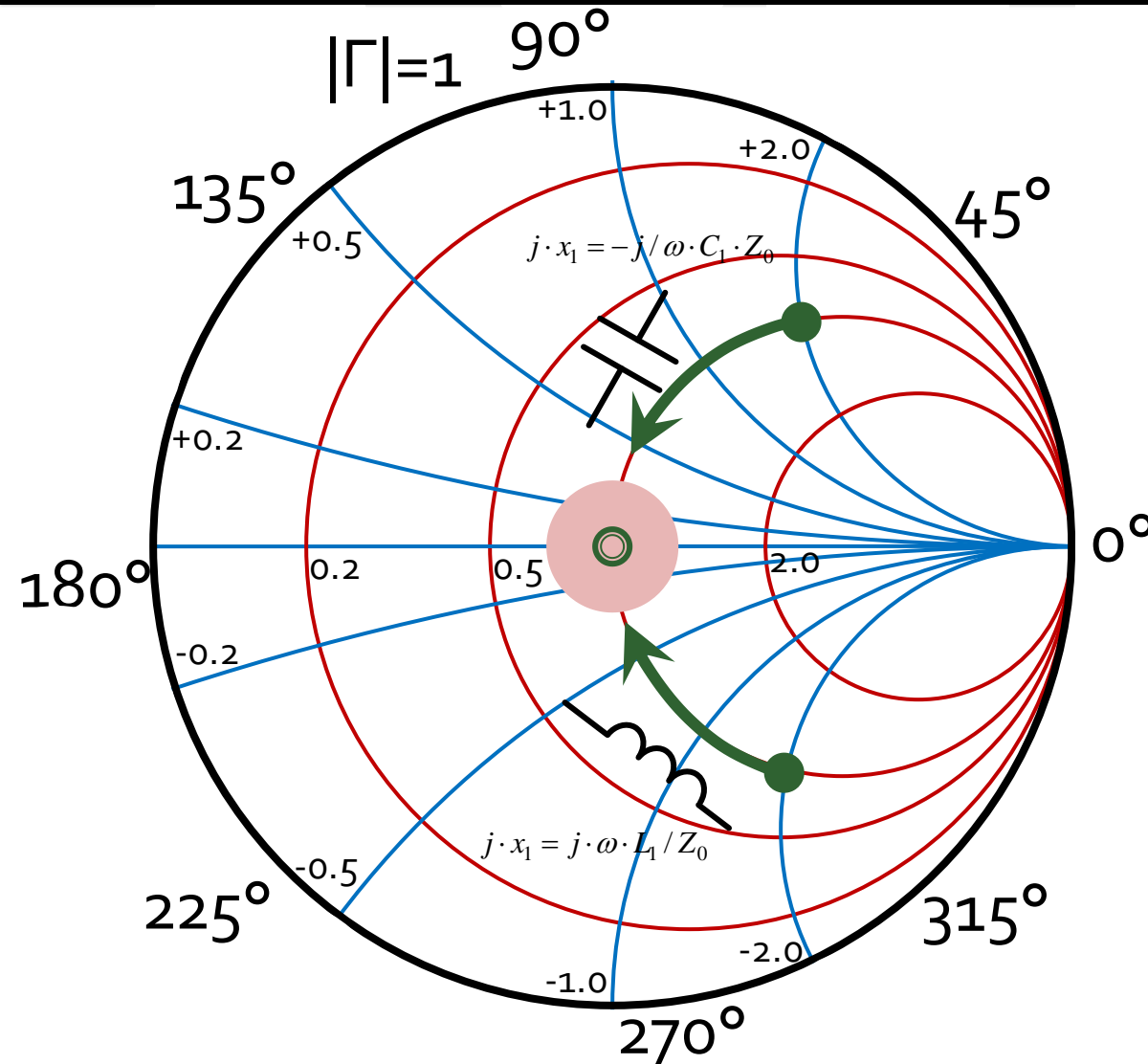
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

Matching, series reactance



$$z_L = r_L + j \cdot x_L$$

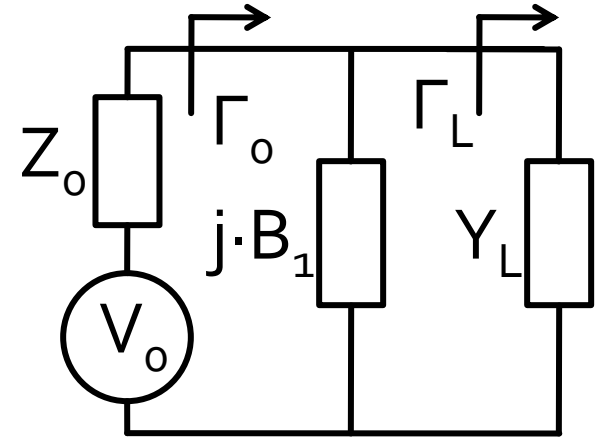
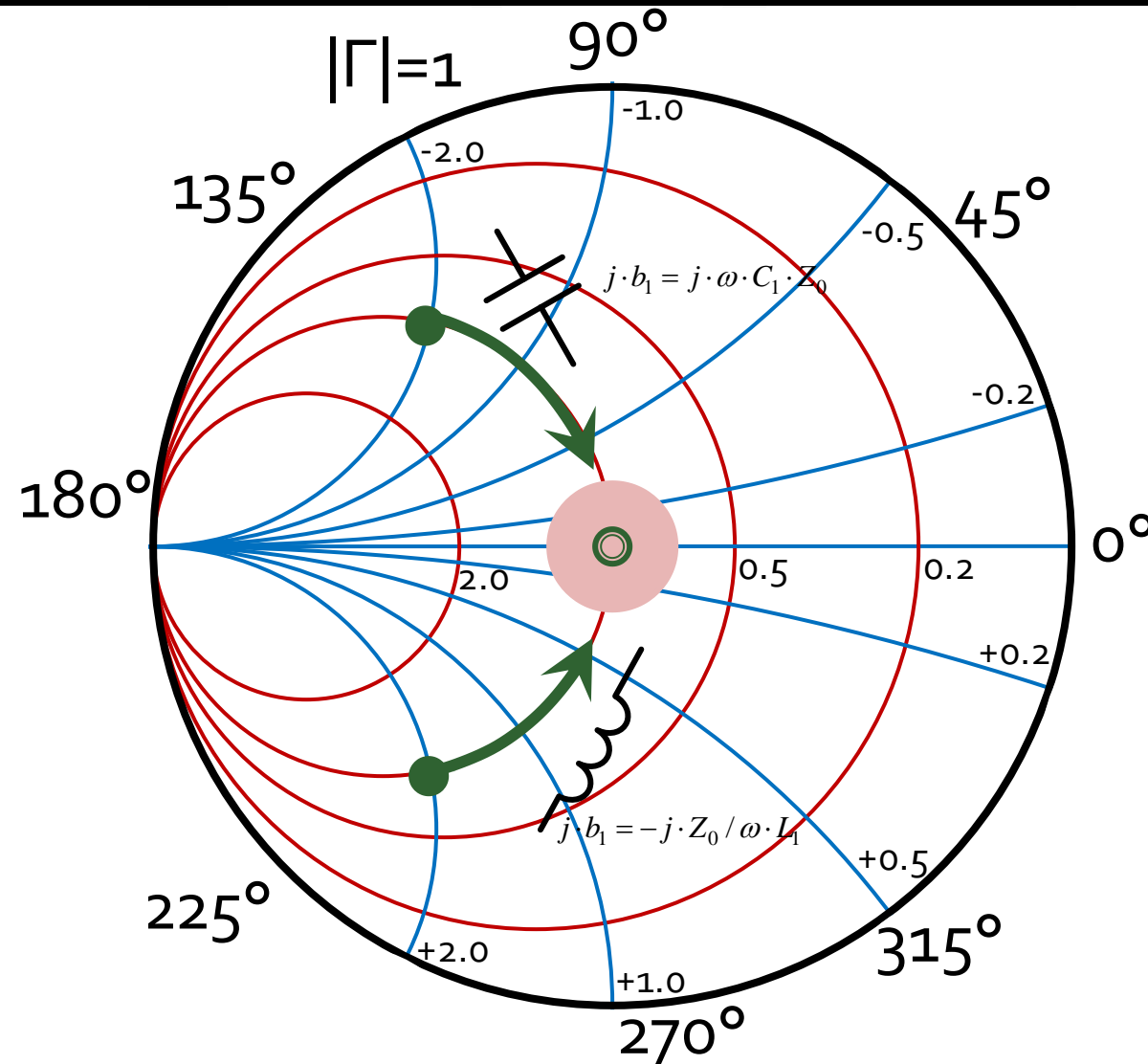
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Matching, shunt susceptance



$$y_L = g_L + j \cdot b_L$$

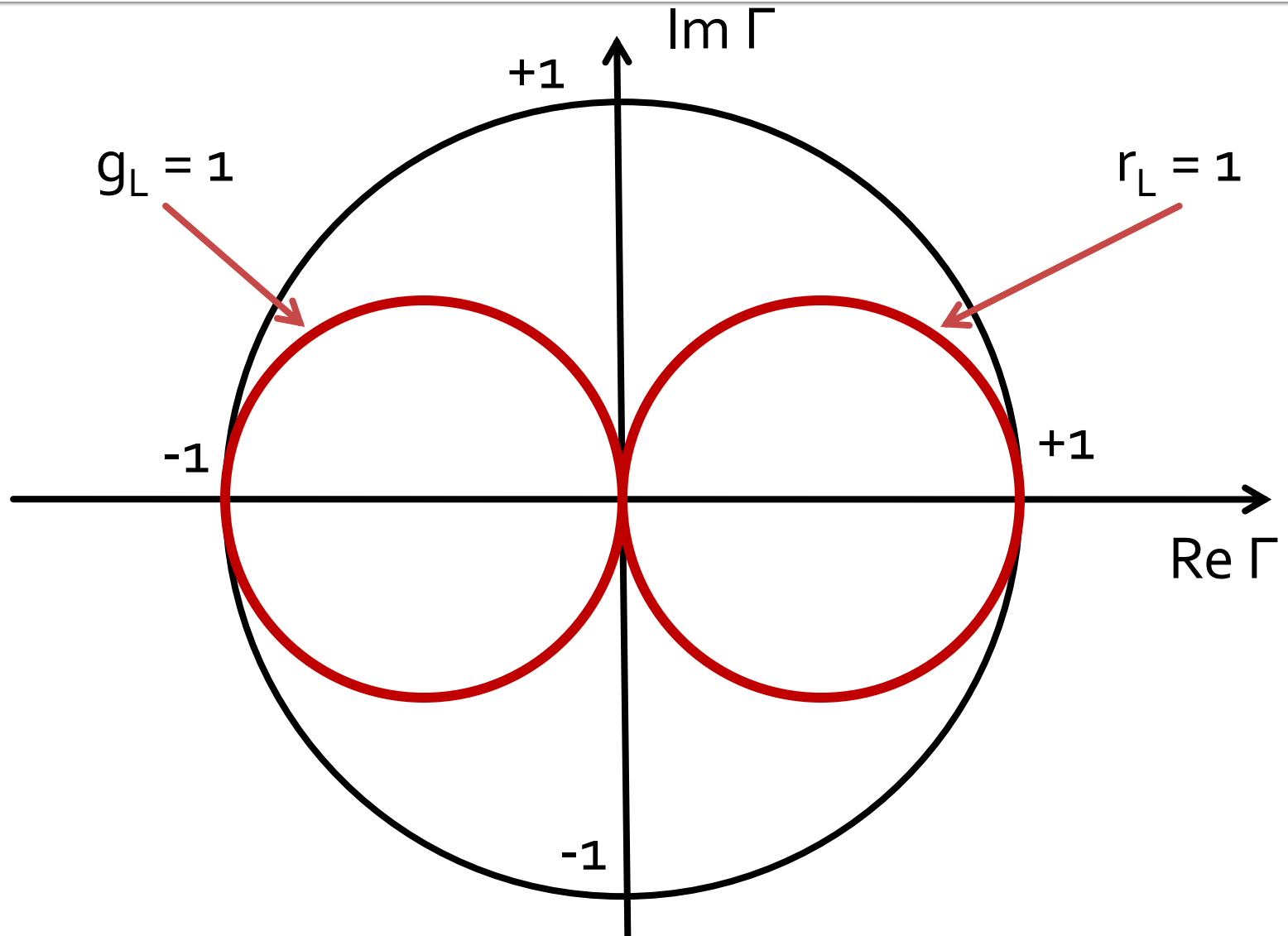
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

- Match can be obtained **if and only if** $g_L = 1$
- we compensate the reactive part of the load

$$j \cdot b_1 = -j \cdot b_L$$

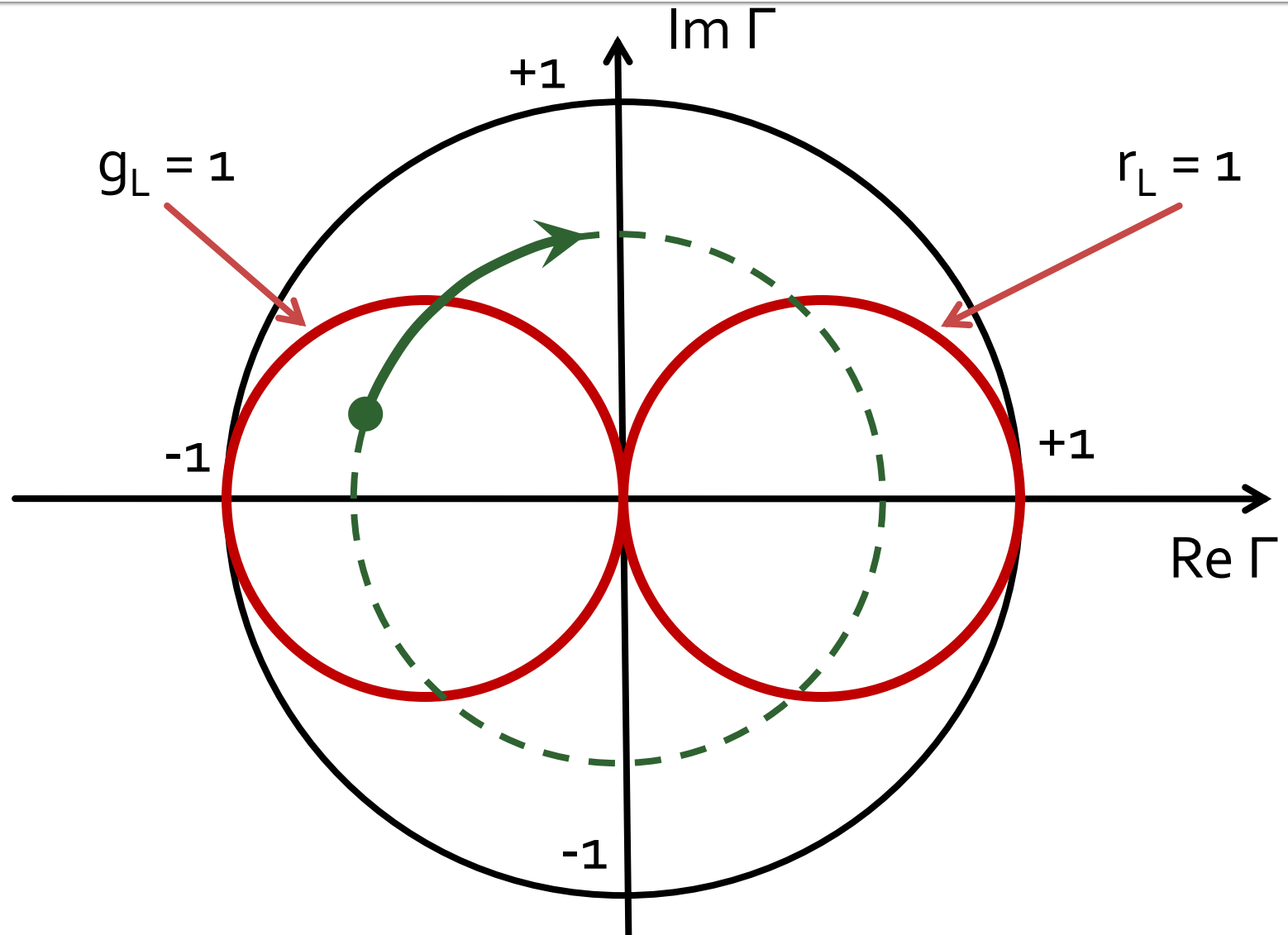
Smith chart, $r=1$ and $g=1$



Impedance Matching

Impedance Matching with Stubs

Smith chart, $r=1$ and $g=1$

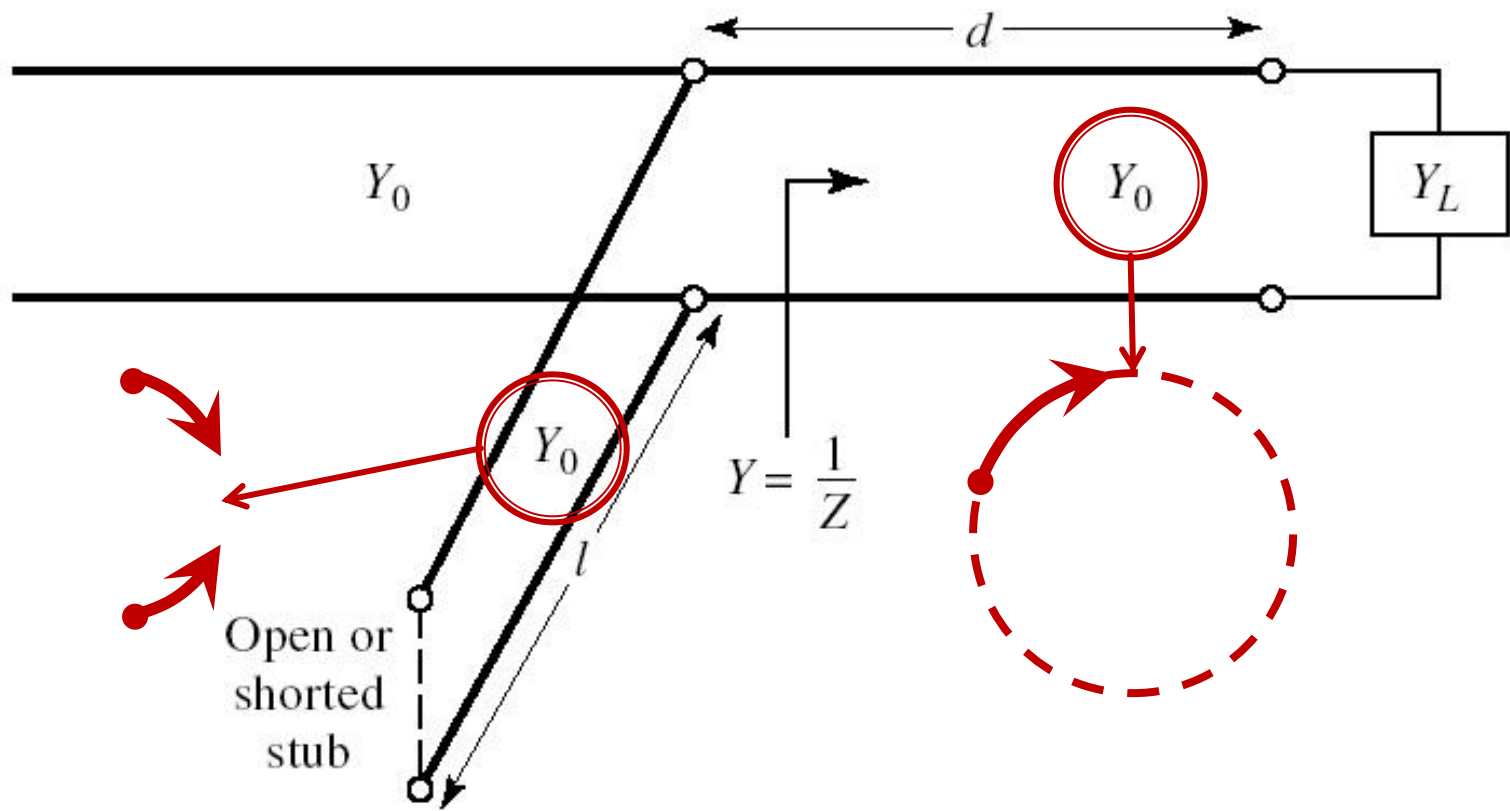


Analytical solutions

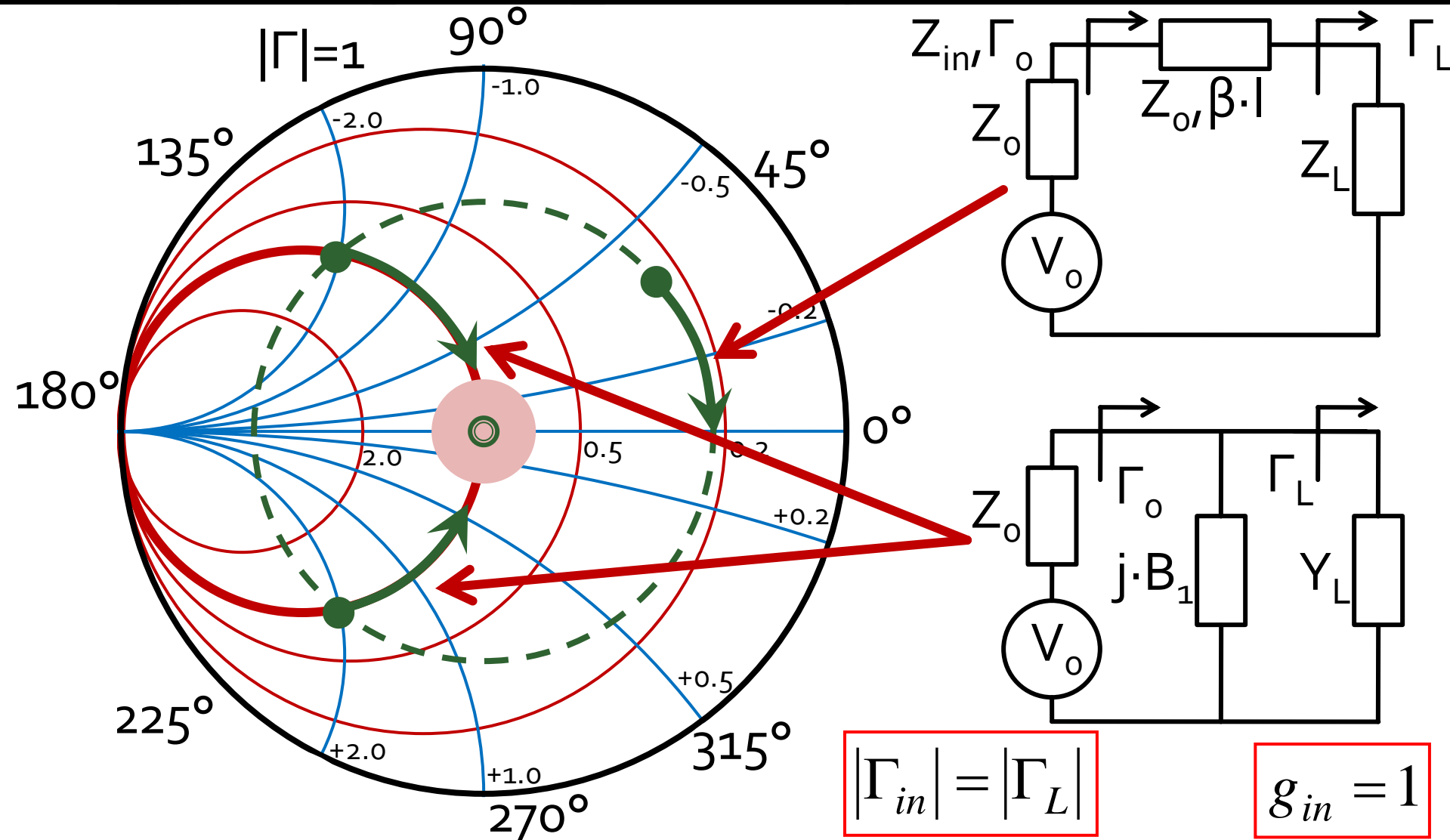
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\Gamma_S = 0.593 \angle 46.85^\circ$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85^\circ \quad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

- **“+” solution** ↓

$$(46.85^\circ + 2\theta) = +126.35^\circ \quad \theta = +39.7^\circ \quad \text{Im } y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

$$\theta_{sp} = \tan^{-1}(\text{Im } y_S) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

- **“-” solution** ↓

$$(46.85^\circ + 2\theta) = -126.35^\circ \quad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\text{Im } y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \quad \theta_{sp} = \tan^{-1}(\text{Im } y_S) = 55.8^\circ$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

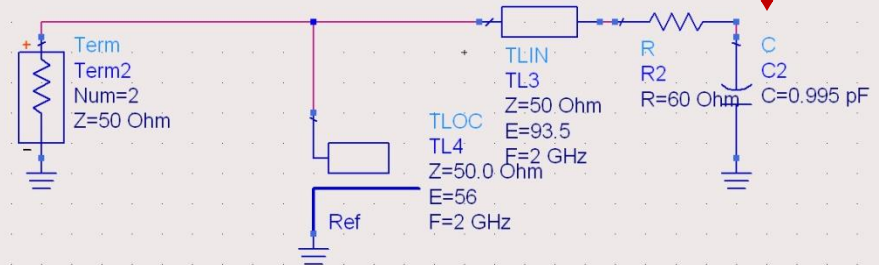
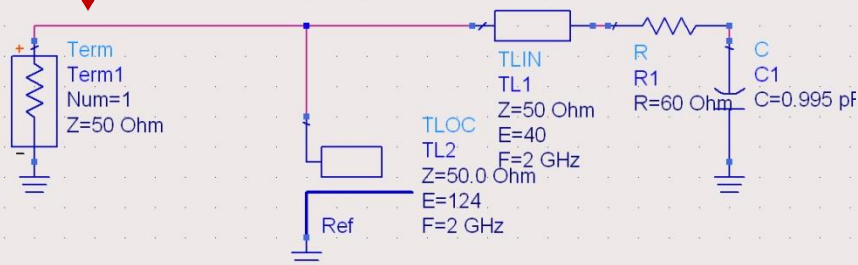
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

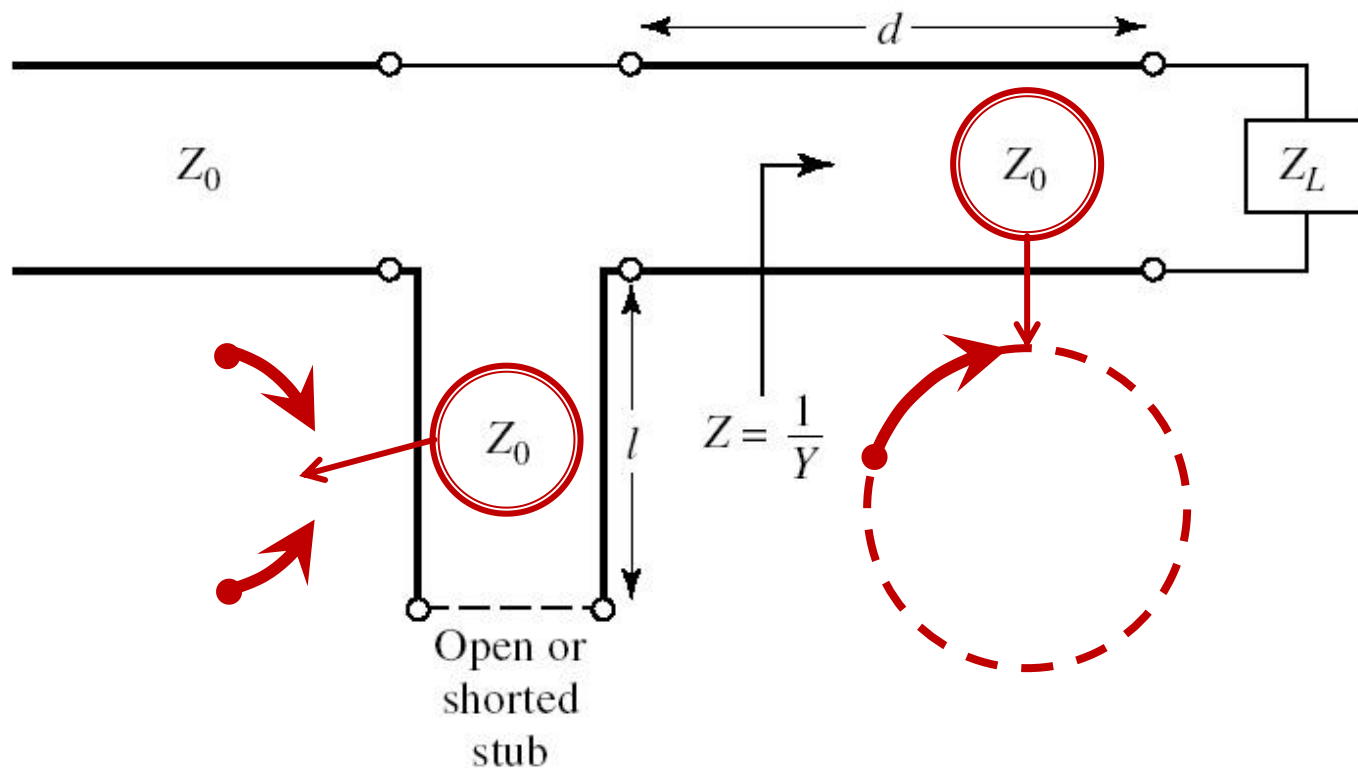
$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

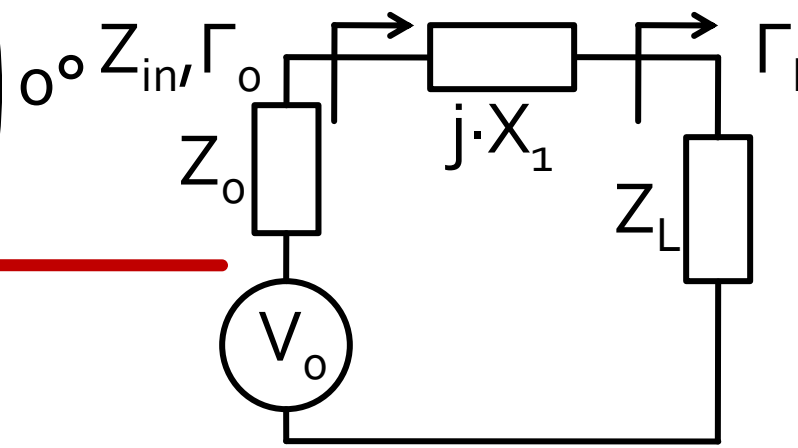
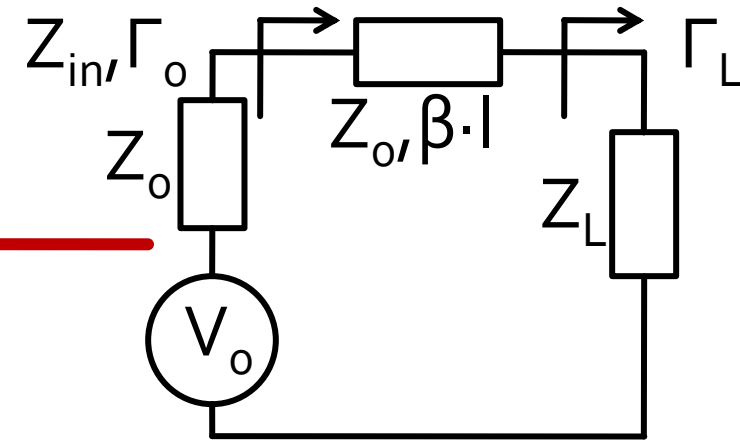
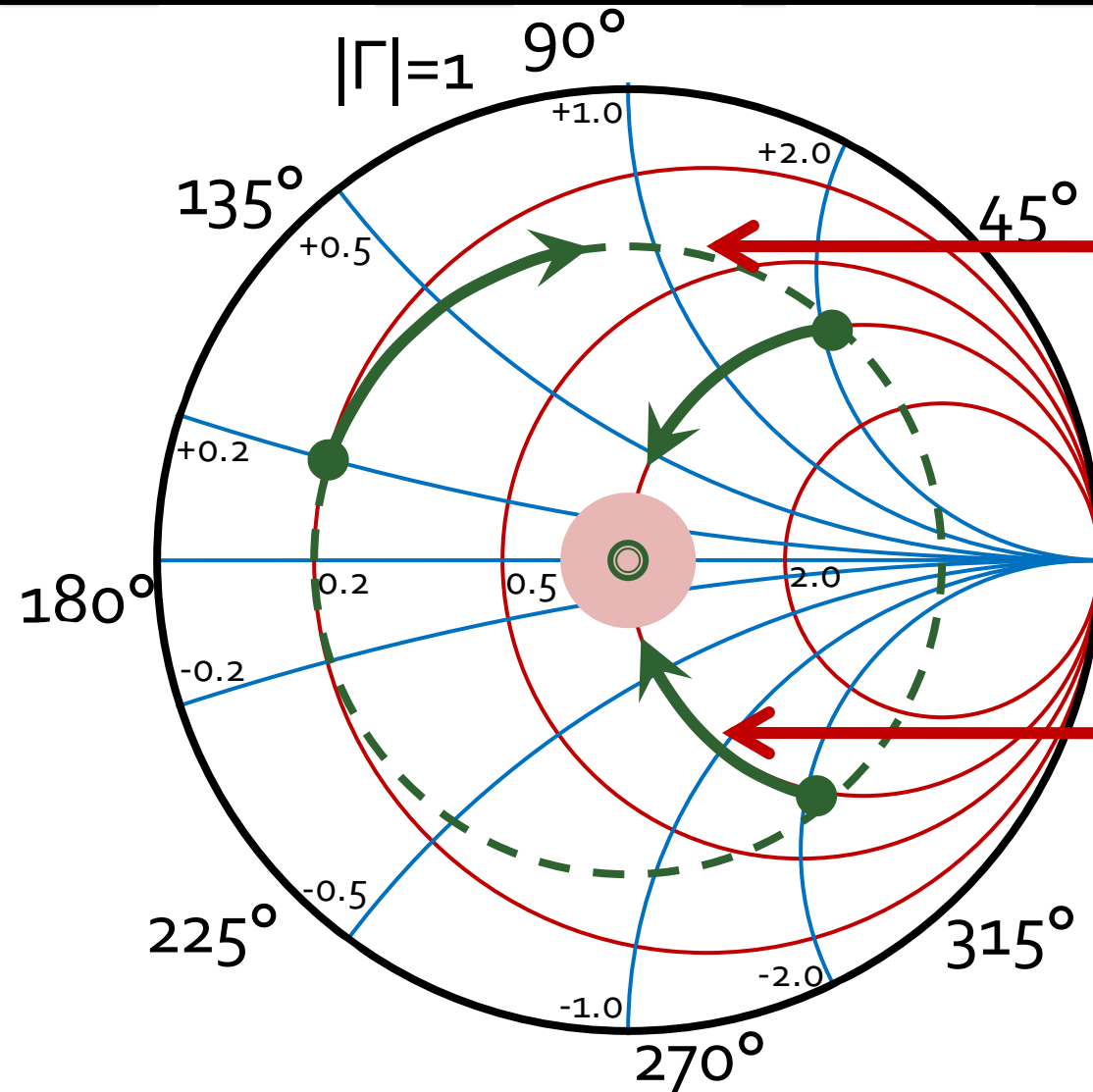


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_s|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

$$\Gamma_s = 0.555 \angle -29.92^\circ$$

$$|\Gamma_s| = 0.555; \quad \varphi = -29.92^\circ \quad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

- **"+" solution** ↓

$$(-29.92^\circ + 2\theta) = +56.28^\circ \quad \theta = 43.1^\circ \quad \text{Im } z_s = \frac{+2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\text{Im } z_s) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

- **"-" solution** ↓

$$(-29.92^\circ + 2\theta) = -56.28^\circ \quad \theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

$$\text{Im } z_s = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = -1.335 \quad \theta_{ss} = -\cot^{-1}(\text{Im } z_s) = 36.8^\circ$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +56.28^\circ \\ -56.28^\circ \end{cases} \quad \theta = \begin{cases} 43.1^\circ \\ 166.8^\circ \end{cases} \quad \text{Im}[z_s(\theta)] = \begin{cases} +1.335 \\ -1.335 \end{cases} \quad \theta_{ss} = \begin{cases} -36.8^\circ + 180^\circ = 143.2^\circ \\ +36.8^\circ \end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

$$l_1 = \frac{43.1^\circ}{360^\circ} \cdot \lambda = 0.120 \cdot \lambda$$

$$l_2 = \frac{143.2^\circ}{360^\circ} \cdot \lambda = 0.398 \cdot \lambda$$

$$l_1 = \frac{166.8^\circ}{360^\circ} \cdot \lambda = 0.463 \cdot \lambda$$

$$l_2 = \frac{36.8^\circ}{360^\circ} \cdot \lambda = 0.102 \cdot \lambda$$



Stub, observations

- adding or subtracting **180°** ($\lambda/2$) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \quad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

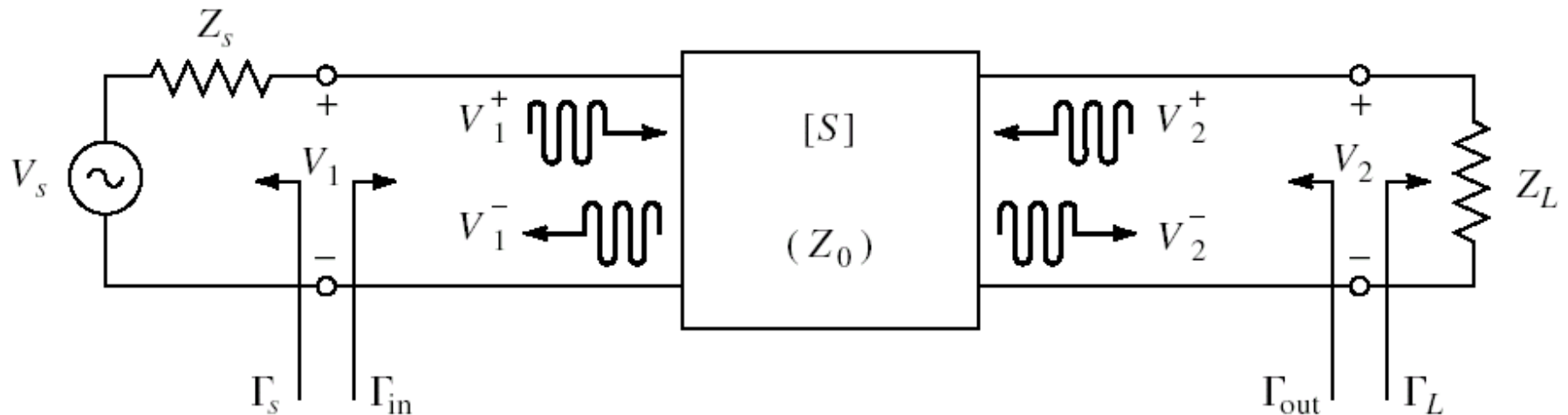
- if the lines/stubs result with **negative** "length"/ "electrical length" we add $\lambda/2$ / 180° to obtain physically realizable lines
- adding or subtracting **90°** ($\lambda/4$) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

- for the stub we can add or subtract 90° ($\lambda/4$) while in the same time changing **open-circuit** \Leftrightarrow **short-circuit**

Microwave Amplifiers

Amplifier as two-port



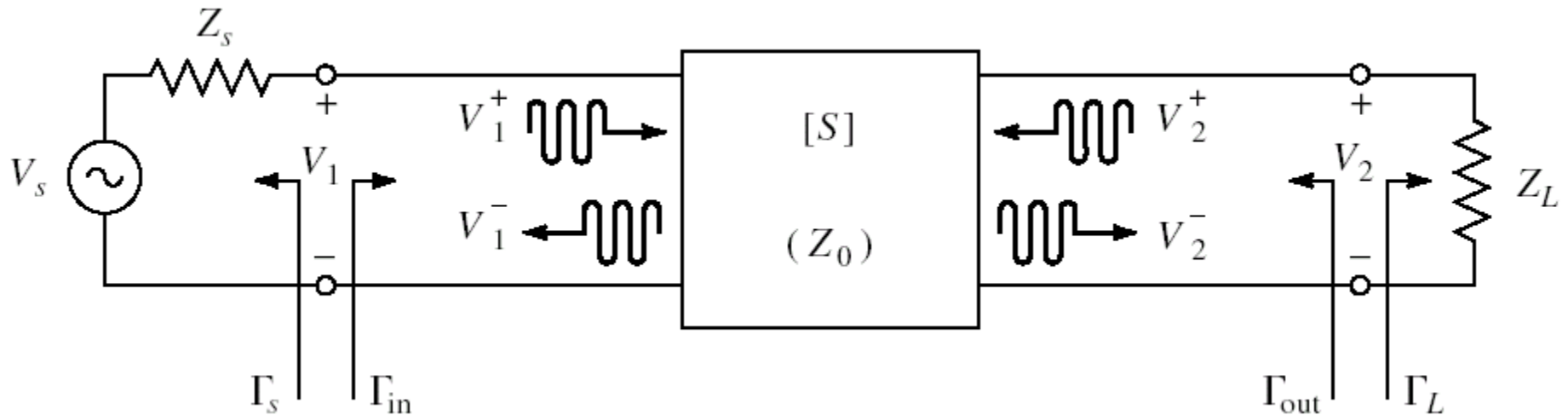
- Charaterized with S parameters
- normalized at Z_0 (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V   ID = 15 mA
# GHz S MA R 50
! f      S11      S21      S12      S22
! GHz  MAG ANG  MAG ANG  MAG ANG  MAG ANG
1.000 0.9800 -18.0 2.230 157.0 0.0240 74.0 0.6900 -15.0
2.000 0.9500 -39.0 2.220 136.0 0.0450 57.0 0.6600 -30.0
3.000 0.8900 -64.0 2.210 110.0 0.0680 40.0 0.6100 -45.0
4.000 0.8200 -89.0 2.230 86.0 0.0850 23.0 0.5600 -62.0
5.000 0.7400 -115.0 2.190 61.0 0.0990 7.0 0.4900 -80.0
6.000 0.6500 -142.0 2.110 36.0 0.1070 -10.0 0.4100 -98.0
!
! f      Fmin  Gammaopt rn/50
! GHz    dB   MAG ANG  -
2.000    1.00 0.72 27 0.84
4.000    1.40 0.64 61 0.58
```

Amplifier as two-port



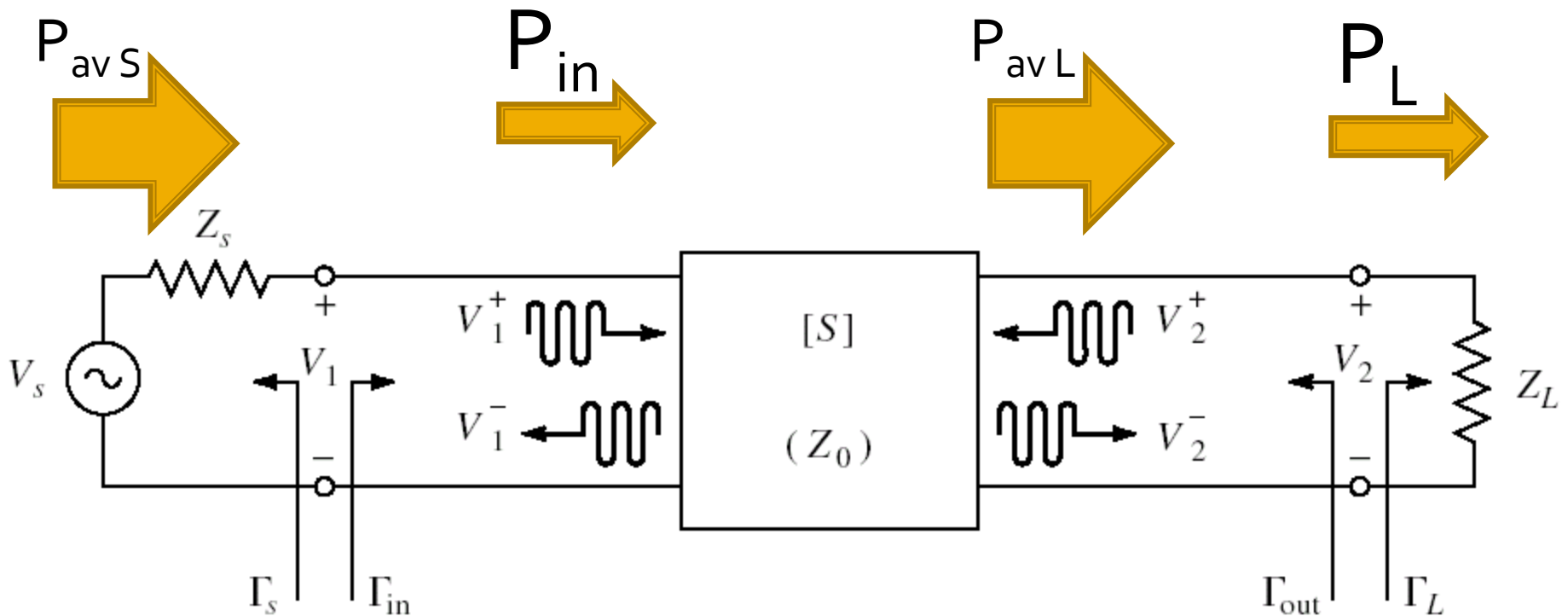
$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_s}{1 - S_{11} \cdot \Gamma_s}$$

Continue

Power / Matching

- Two ports in which matching influences the power transfer



Signal power

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1 = \frac{V_S \cdot Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in})$$

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \cdot \Gamma_{in})}$$

■ L3
$$P_{in} = \frac{1}{2 \cdot Z_0} \cdot |V_1^+|^2 \cdot (1 - |\Gamma_{in}|^2)$$

$$P_L = \frac{1}{2 \cdot Z_0} \cdot |V_2^-|^2 \cdot (1 - |\Gamma_L|^2)$$

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = \frac{S_{21} \cdot V_1^+}{1 - S_{22} \cdot \Gamma_L}$$

$$P_L = \frac{|V_1^+|^2}{2 \cdot Z_0} \cdot \frac{|S_{21}|^2}{|1 - S_{22} \cdot \Gamma_L|^2} (1 - |\Gamma_L|^2)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

Signal power

- Signal power

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{|1 - S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \cdot \Gamma_{in}|^2}$$

- Power available from the source

$$P_{av S} = P_{in}|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1 - \Gamma_S|^2}{(1 - |\Gamma_S|^2)}$$

- Power available on the load (from the network)

$$P_{av L} = P_L|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot |1 - \Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

Two-Port Power Gains

- Power Gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2) \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$P_{in} = P_{in}(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

$$P_L = P_L(\Gamma_S, \Gamma_{in}(\Gamma_L), S)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av L}}{P_{av S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2)}{|1 - S_{22} \cdot \Gamma_L|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2) \cdot (1 - |\Gamma_L|^2)}{|1 - \Gamma_S \cdot \Gamma_{in}|^2 \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

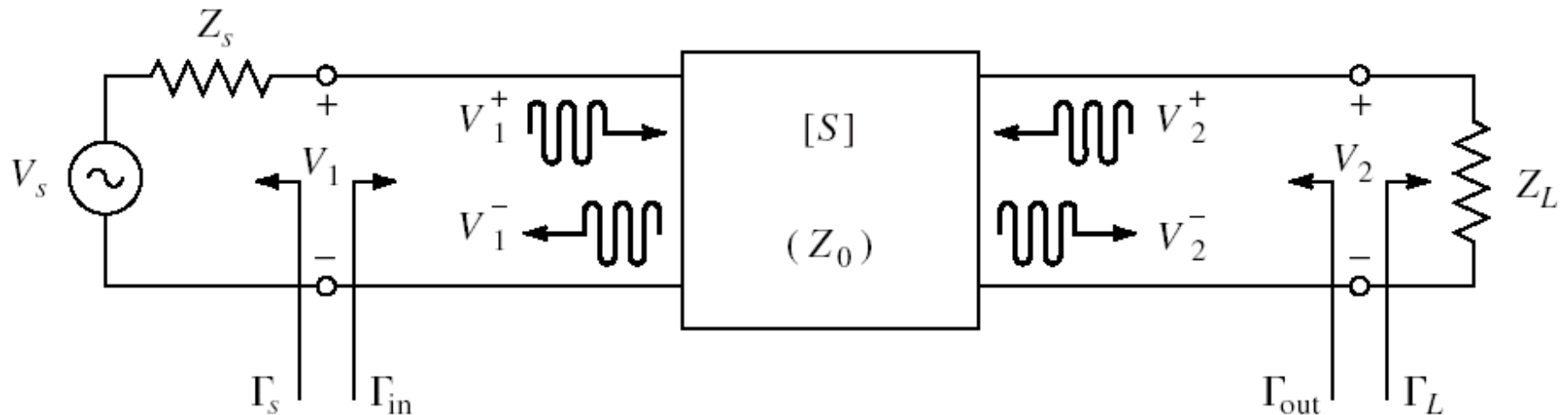
$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$S_{12} \cong 0$$

$$\Gamma_{in} = S_{11}$$

Input and output can be treated independently

Amplifier as two-port

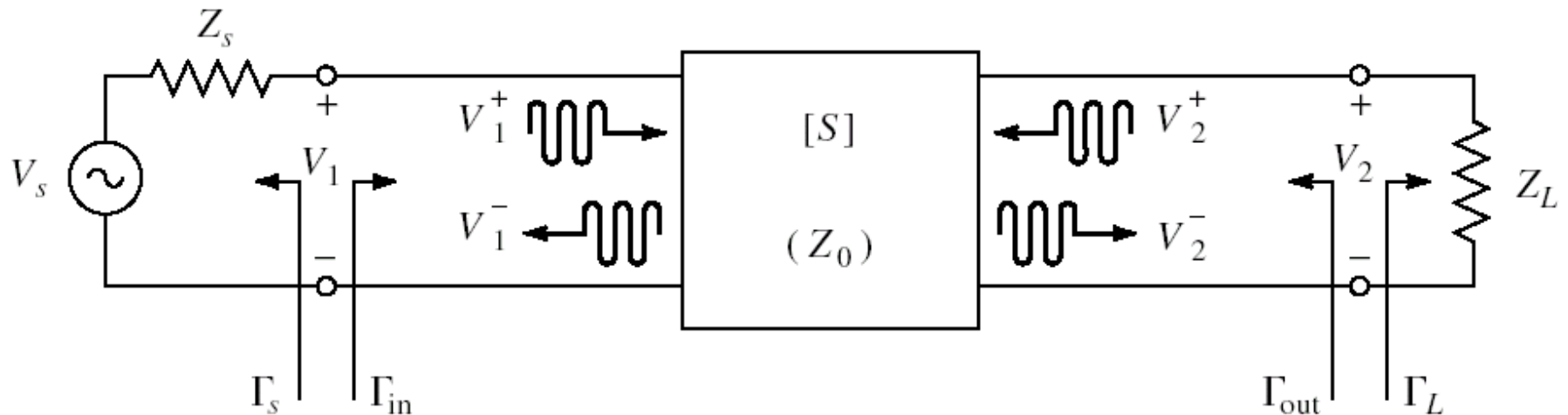


- For an amplifier two-port we are interested in:
 - stability
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Microwave Amplifiers

Stability

Amplifier as two-port



- For an amplifier two-port we are interested in:
 - **stability**
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Stability

- L7 $\Gamma = \Gamma_r + j \cdot \Gamma_i$ $r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$
 Z_{in} $\Gamma_{in} = \Gamma_r + j \cdot \Gamma_i$
- instability
 $\text{Re}\{Z_{in}\} < 0 \iff 1 - \Gamma_r^2 - \Gamma_i^2 < 0 \quad \Gamma_r^2 + \Gamma_i^2 > 1 \quad |\Gamma_{in}| > 1$
- stability, Z_{in}
 - conditions to be met by Γ_L to achieve (input) stability
 $|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$
- similarly Z_{out}
 - conditions to be met by Γ_S to achieve (output) stability

Stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- We can calculate conditions to be met by Γ_L to achieve stability

$$|\Gamma_{out}| < 1 \quad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| < 1$$

- We can calculate conditions to be met by Γ_S to achieve stability

Stability

$$|\Gamma_{in}| < 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \quad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$|S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

- determinant of the S matrix $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

$$|S_{11} - \Delta \cdot \Gamma_L| = |1 - S_{22} \cdot \Gamma_L|$$

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

Stability

$$|S_{11} - \Delta \cdot \Gamma_L|^2 = |1 - S_{22} \cdot \Gamma_L|^2$$

$$a \cdot a^* = |a| \cdot e^{j\theta} \cdot |a| \cdot e^{-j\theta} = |a|^2$$

$$|a + b|^2 = (a + b) \cdot (a + b)^* = (a + b) \cdot (a^* + b^*) = \underbrace{|a|^2} + \underbrace{|b|^2} + \underbrace{a^* \cdot b + a \cdot b^*}$$

$$|S_{11}|^2 + |\Delta|^2 \cdot |\Gamma_L|^2 - (\Delta \cdot \Gamma_L \cdot S_{11}^* + \Delta^* \cdot \Gamma_L^* \cdot S_{11}) = 1 + |S_{22}|^2 \cdot |\Gamma_L|^2 - (S_{22}^* \cdot \Gamma_L^* + S_{22} \cdot \Gamma_L)$$

$$(|S_{22}|^2 - |\Delta|^2) \cdot \Gamma_L \cdot \Gamma_L^* - (S_{22} - \Delta \cdot S_{11}^*) \cdot \Gamma_L - (S_{22}^* - \Delta^* \cdot S_{11}) \cdot \Gamma_L^* = |S_{11}|^2 - 1$$

$$\Gamma_L \cdot \Gamma_L^* - \frac{(S_{22} - \Delta \cdot S_{11}^*) \cdot \Gamma_L + (S_{22}^* - \Delta^* \cdot S_{11}) \cdot \Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{|S_{22} - \Delta \cdot S_{11}^*|^2}{(|S_{22}|^2 - |\Delta|^2)^2}$$

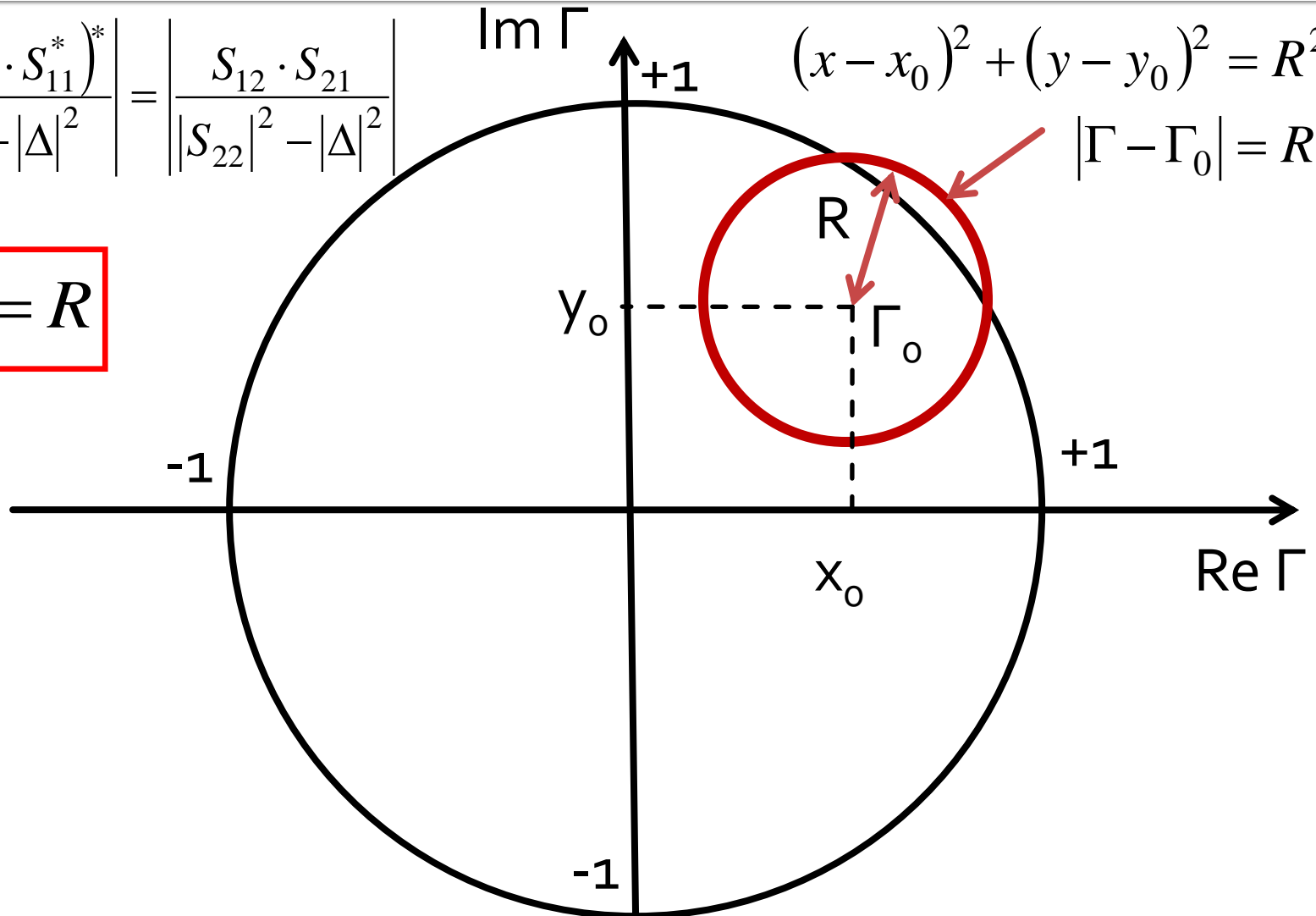
$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right|^2 = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{|S_{22} - \Delta \cdot S_{11}^*|^2}{(|S_{22}|^2 - |\Delta|^2)^2}$$

Stability

$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$(x - x_0)^2 + (y - y_0)^2 = R^2$
 $|\Gamma - \Gamma_0| = R$

$$|\Gamma - \Gamma_0| = R$$



Output stability circle (CSOUT)

$$\left| \Gamma_L - \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad R_L = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{22}|^2 - |\Delta|^2 \right|}$$

Input stability circle (CSIN)

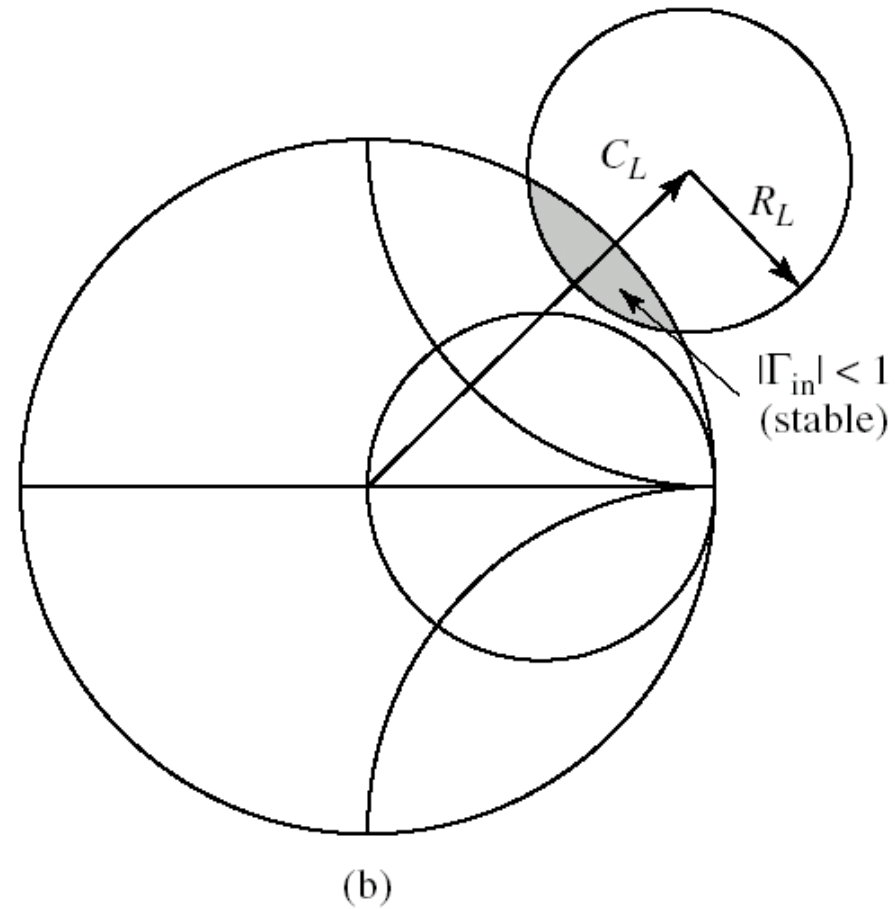
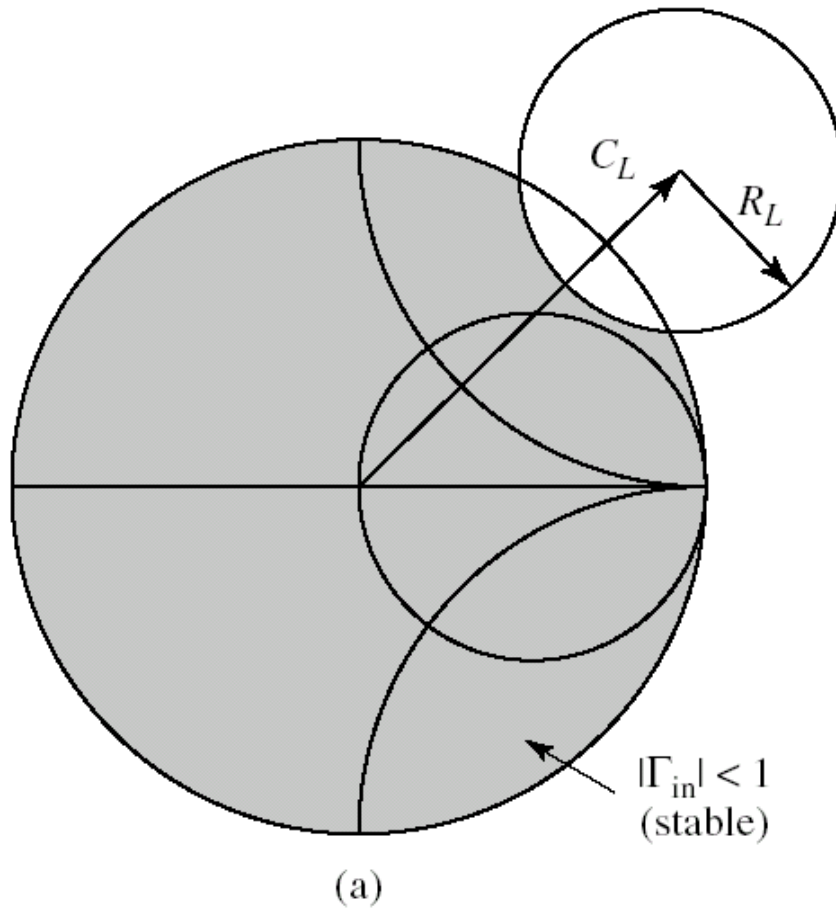
- Similarly $|\Gamma_{out}| = 1$ $\left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_S for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** (Γ_S)

$$C_S = \frac{(S_{11} - \Delta \cdot S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad R_S = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{11}|^2 - |\Delta|^2 \right|}$$

Output stability circle (CSOUT)

- The **output stability circle** represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- The circle divides the complex planes in two areas, the **inside** and the **outside** of the circle
- The two areas will represent the locus of Γ_L for stability ($|\Gamma_{in}| < 1$) / instability ($|\Gamma_{in}| > 1$)

Output stability circle (CSOUT)



- Two cases possible: (a) stable outside/ (b) stable inside

Output stability circle (CSOUT)

- Identification of the stability / instability regions
 - The center of the Smith Chart in Γ_L complex plane corresponds to $\Gamma_L = 0$
 - Input reflection coefficient

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \quad \Gamma_{in}|_{\Gamma_L=0} = S_{11} \quad |\Gamma_{in}|_{\Gamma_L=0} = |S_{11}|$$

- A decision can be made based on $|S_{11}|$ value and on the position of the center of the Smith chart (origin of the complex plane) relative to the circle

Identification of the stability / instability regions

- Output stability circle
 - $|S_{11}| < 1 \rightarrow$ the center of the Smith chart on which Γ_L is represented is a **stable point**, so it's placed in the stability region (most often situation)
 - $|S_{11}| > 1 \rightarrow$ the center of the Smith chart on which Γ_L is represented is a **unstable point**, so it's placed in the instability region
- Input stability circle
 - $|S_{22}| < 1 \rightarrow$ the center of the Smith chart on which Γ_S is represented is a **stable point**, so it's placed in the stability region (most often situation)
 - $|S_{22}| > 1 \rightarrow$ the center of the Smith chart on which Γ_S is represented is a **unstable point**, so it's placed in the instability region

Example

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @5GHz
 - $S_{11} = 0.64 \angle 139^\circ$
 - $S_{12} = 0.119 \angle -21^\circ$
 - $S_{21} = 3.165 \angle 16^\circ$
 - $S_{22} = 0.22 \angle 146^\circ$



```

!ATF-34143
!S-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99

# ghz s ma r 50

2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46

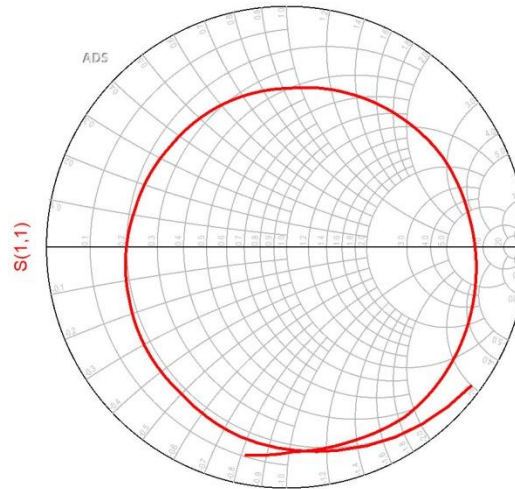
!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -

2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
    
```

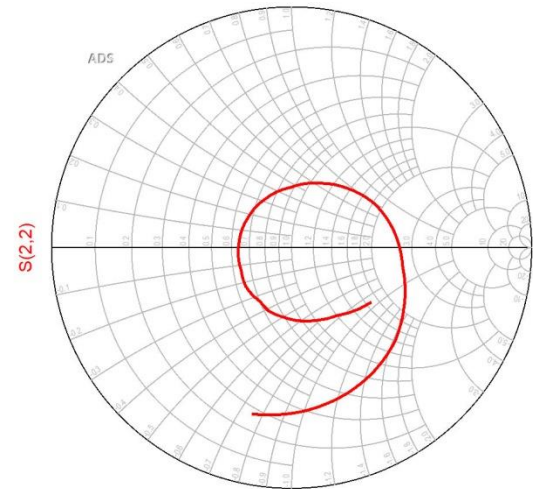
$$\begin{cases}
 S_{11} = 0.64 \angle 139^\circ \\
 S_{11} = 0.64 \cdot \cos 139^\circ + j \cdot 0.64 \cdot \sin 139^\circ \\
 S_{11} = -0.4830 + j \cdot 0.4199
 \end{cases}$$

Example

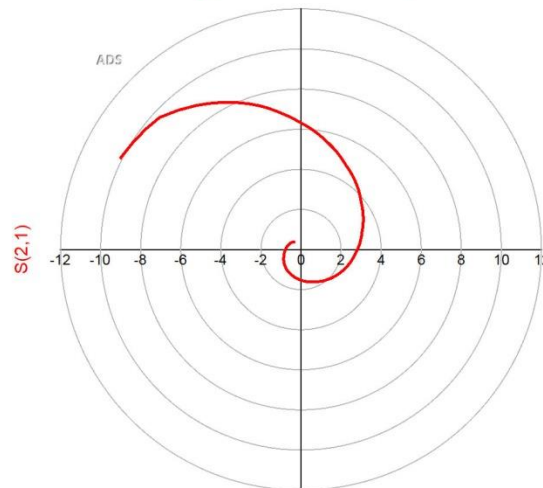
- ATF-34143
- at
 - $V_{ds}=3V$
 - $I_d=20mA$.



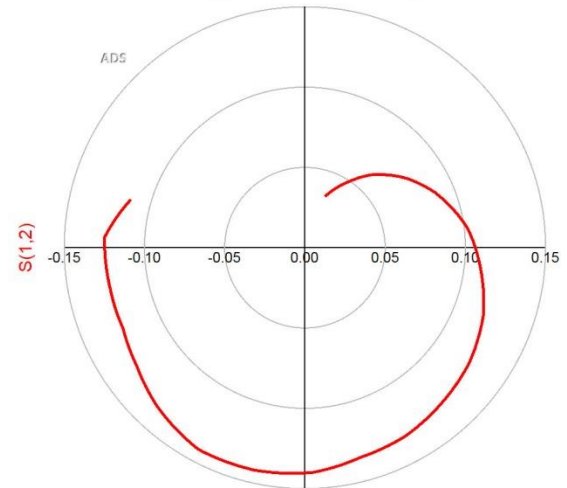
freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)



freq (500.0MHz to 18.00GHz)

Solution + region identification

- S parameters

- $S_{11} = -0.483 + 0.42 \cdot j$

- $S_{12} = 0.111 - 0.043 \cdot j$

- $S_{21} = 3.042 + 0.872 \cdot j$

- $S_{22} = -0.182 + 0.123 \cdot j$

- $|S_{11}| = 0.64 < 1$

- $|C_L| < R_L, o \in \text{CSOUT}$

- The center of the Smith chart is placed inside the output stability circle ($o \in \text{CSOUT}$) and is a stable point ($|S_{11}| < 1$)

- the inside of the output stability circle – stability region

- the outside of the output stability circle – instability region

$$C_L = \frac{(S_{22} - \Delta \cdot S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 3.931 - 0.897 \cdot j$$

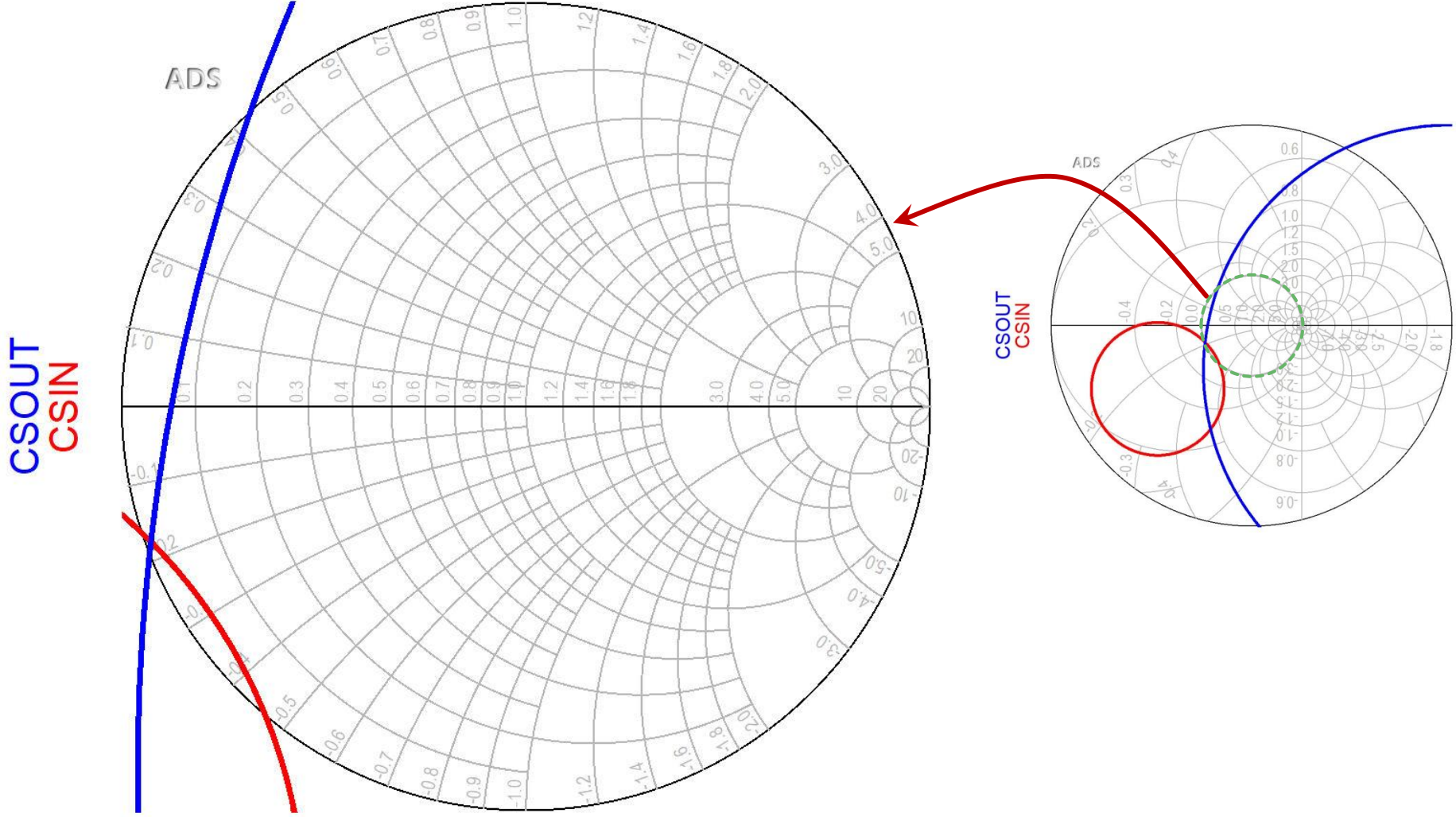
$$|C_L| = 4.032$$

$$R_L = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{22}|^2 - |\Delta|^2 \right|} = 4.891$$

Solution + region identification

- S parameters
 - $S_{11} = -0.483 + 0.42 \cdot j$
 - $S_{12} = 0.111 - 0.043 \cdot j$
 - $S_{21} = 3.042 + 0.872 \cdot j$
 - $S_{22} = -0.182 + 0.123 \cdot j$
 - $|S_{22}| = 0.22 < 1$
 - $|C_S| > R_S, 0 \notin \text{CSIN}$
 - The center of the Smith chart is placed outside the input stability circle ($0 \notin \text{CSIN}$) and is a stable point ($|S_{22}| < 1$)
 - the outside of the input stability circle – stability region
 - the inside of the input stability circle – instability region
- $$C_S = \frac{(S_{11} - \Delta \cdot S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = -1.871 - 1.265 \cdot j$$
- $$|C_S| = 2.259$$
- $$R_S = \frac{|S_{12} \cdot S_{21}|}{\left| |S_{11}|^2 - |\Delta|^2 \right|} = 1.325$$

ADS

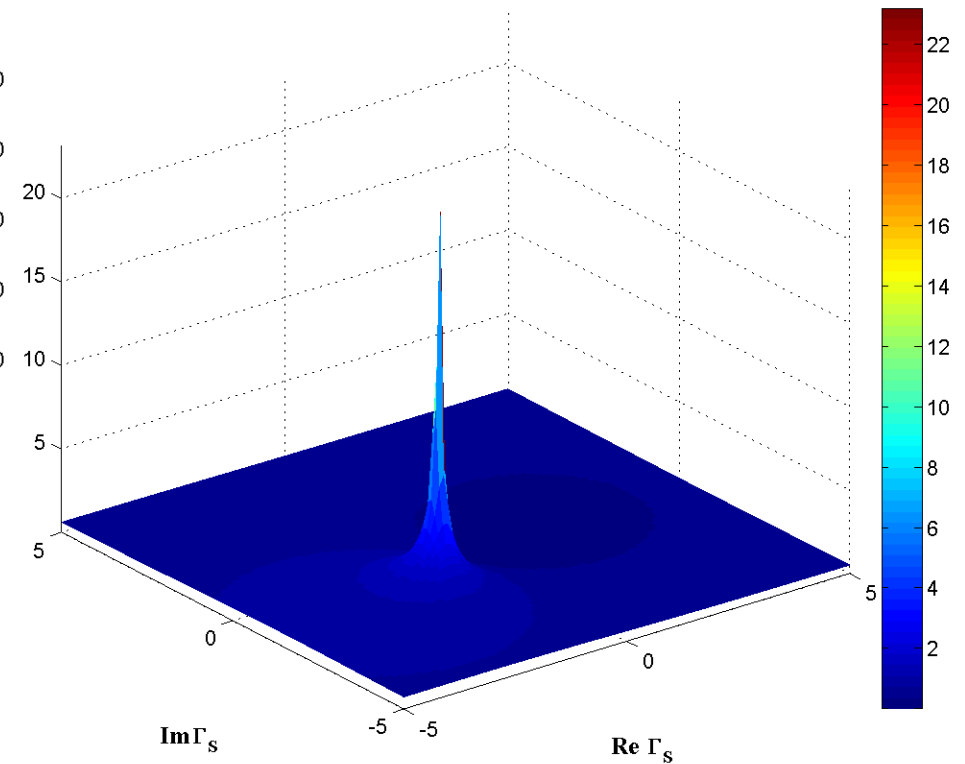
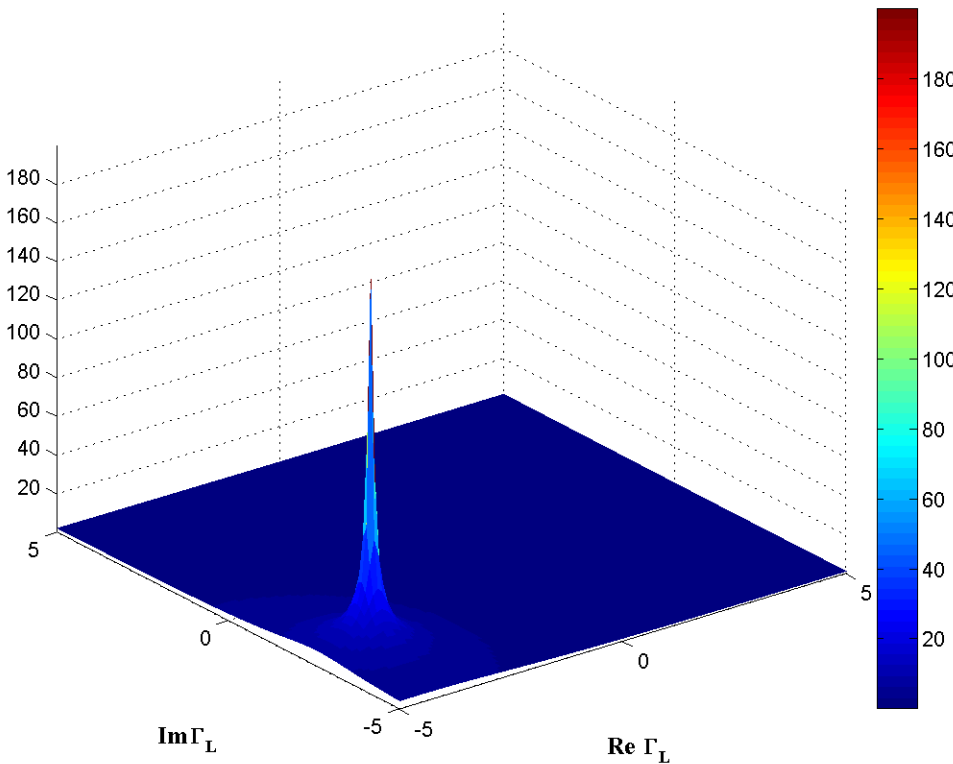


3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- High variations -> we change to z logarithmic scale

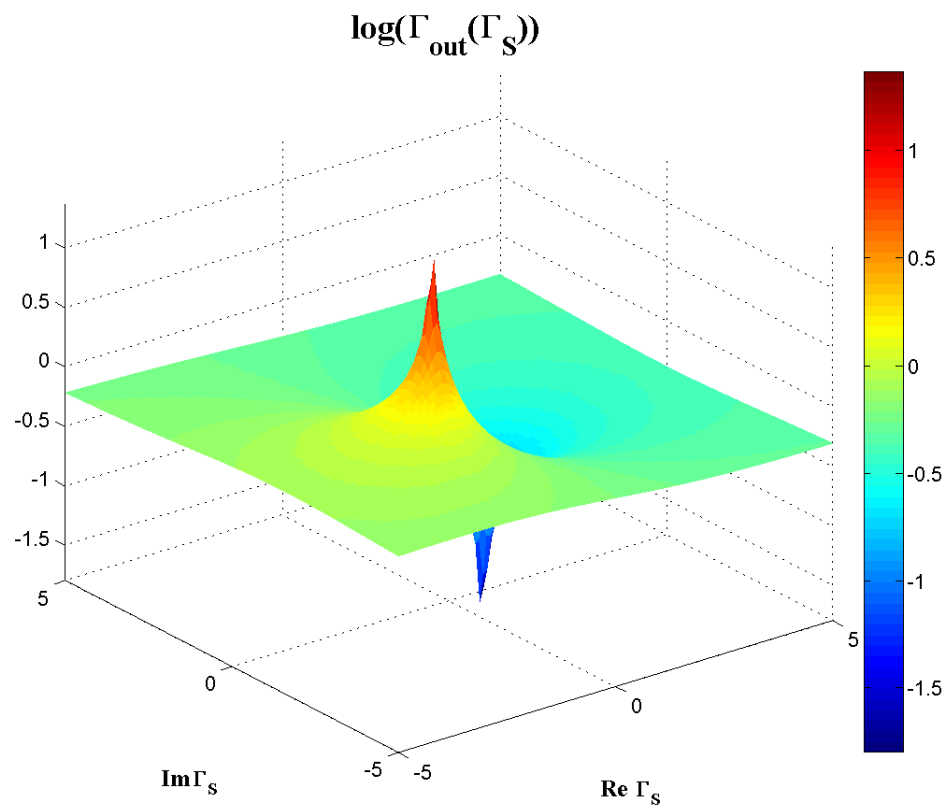
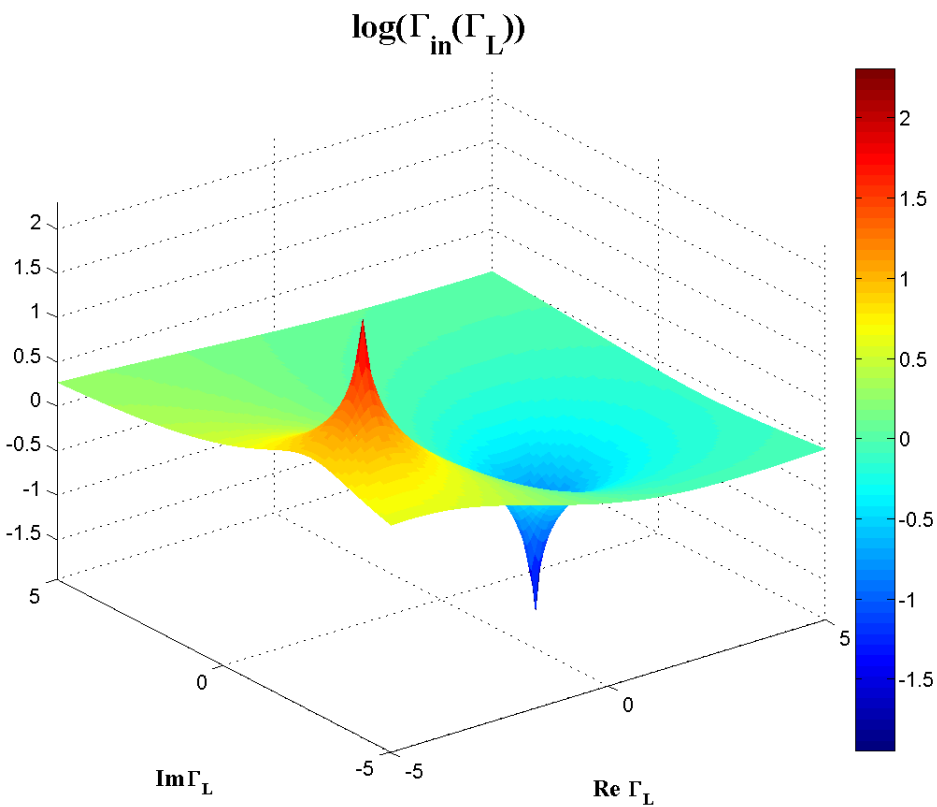
scale $\Gamma_{in}(\Gamma_L) = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$

$\Gamma_{out}(\Gamma_S) = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$



3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

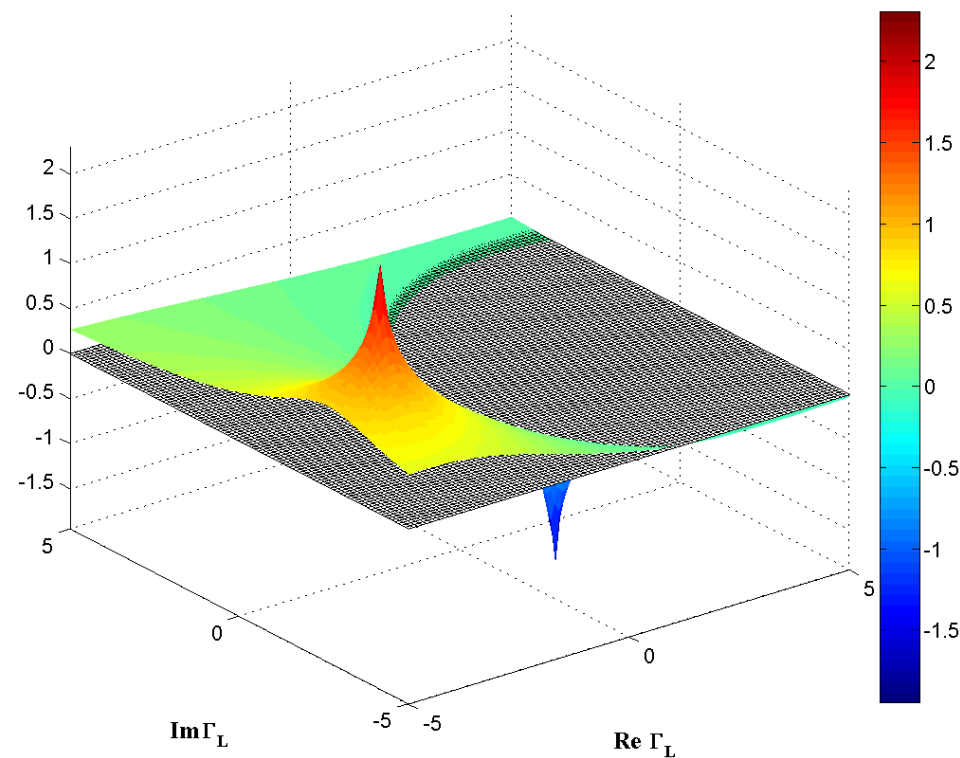
- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$



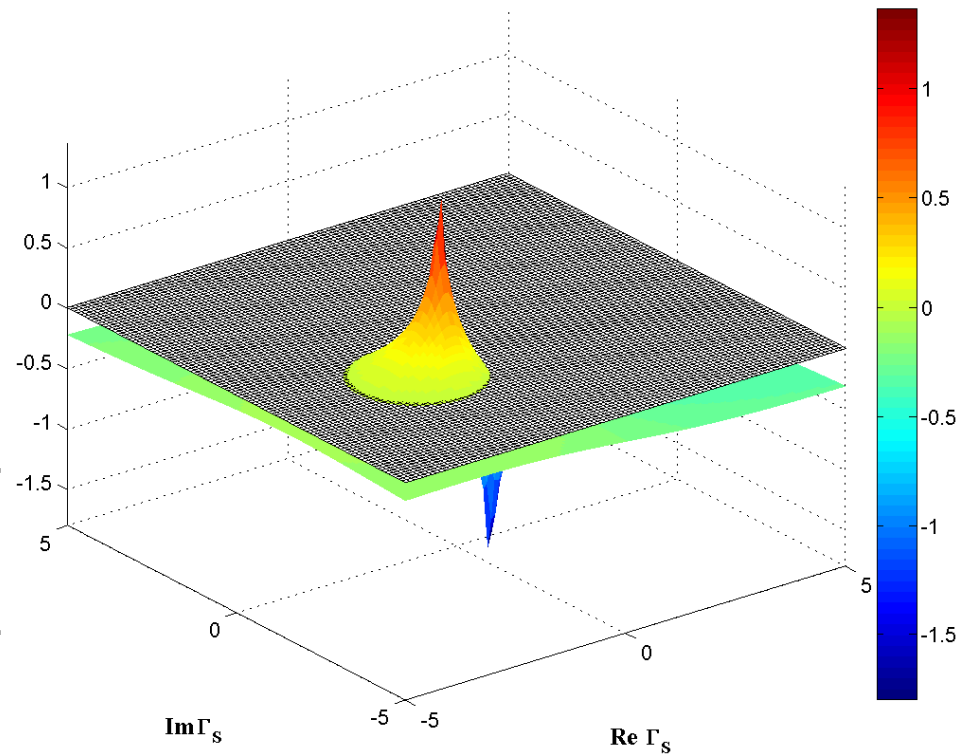
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane $z = 0$ is a circle

$\log(\Gamma_{in}(\Gamma_L))$

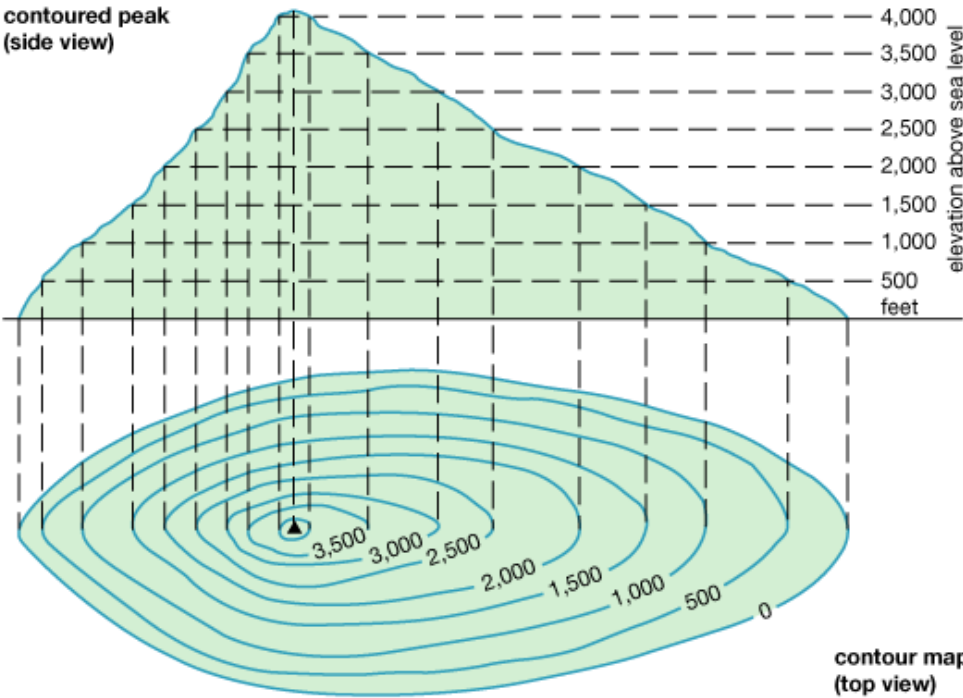


$\log(\Gamma_{out}(\Gamma_S))$

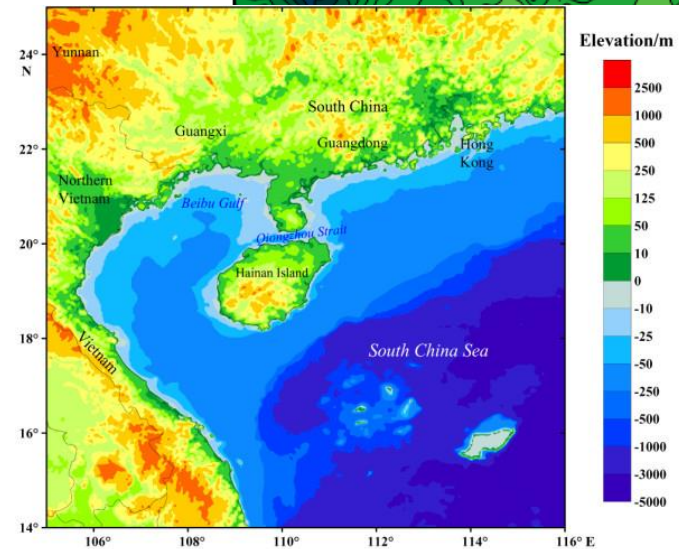
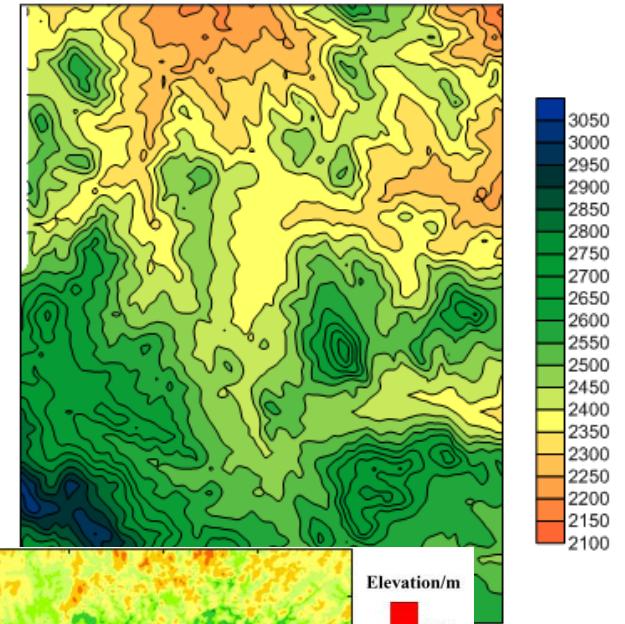


Contour map/lines

contoured peak
(side view)



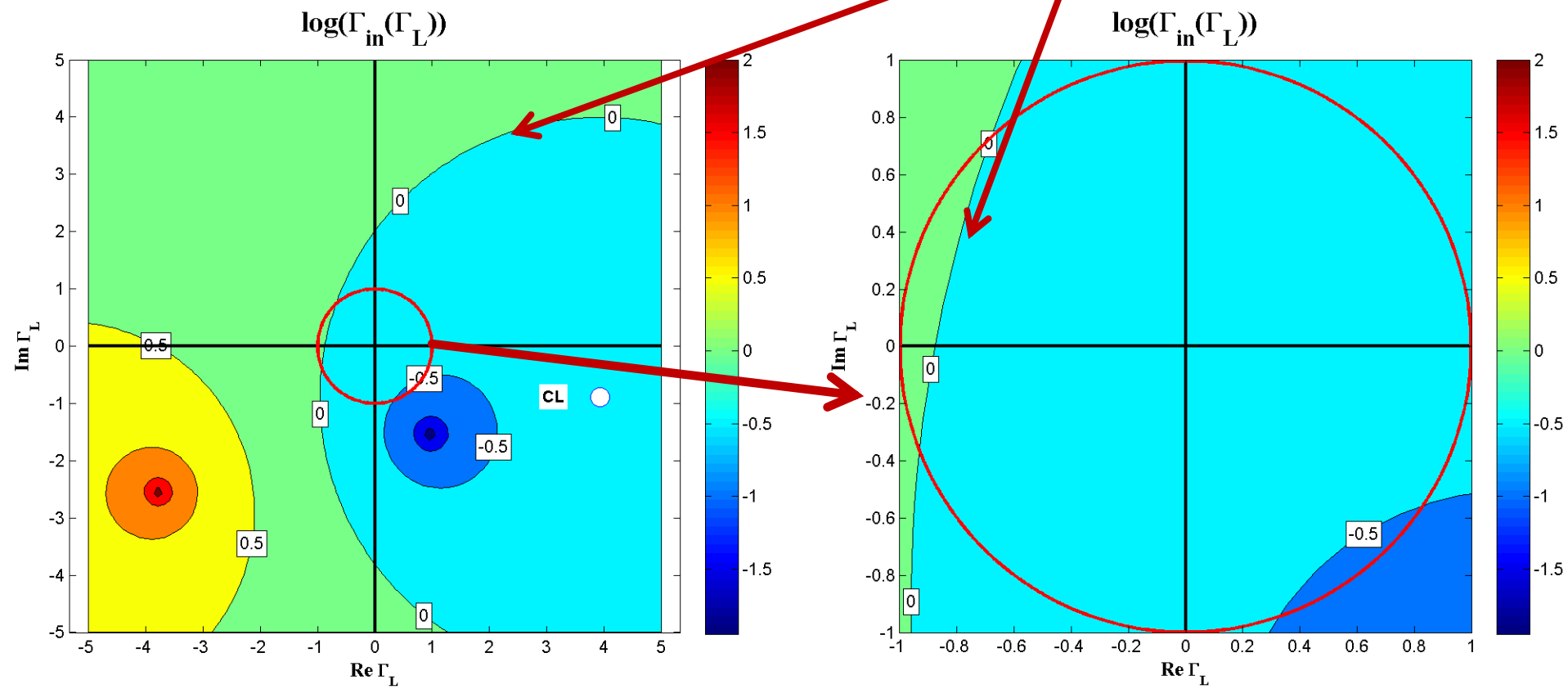
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Contour lines of $\log_{10}|\Gamma_{in}|$

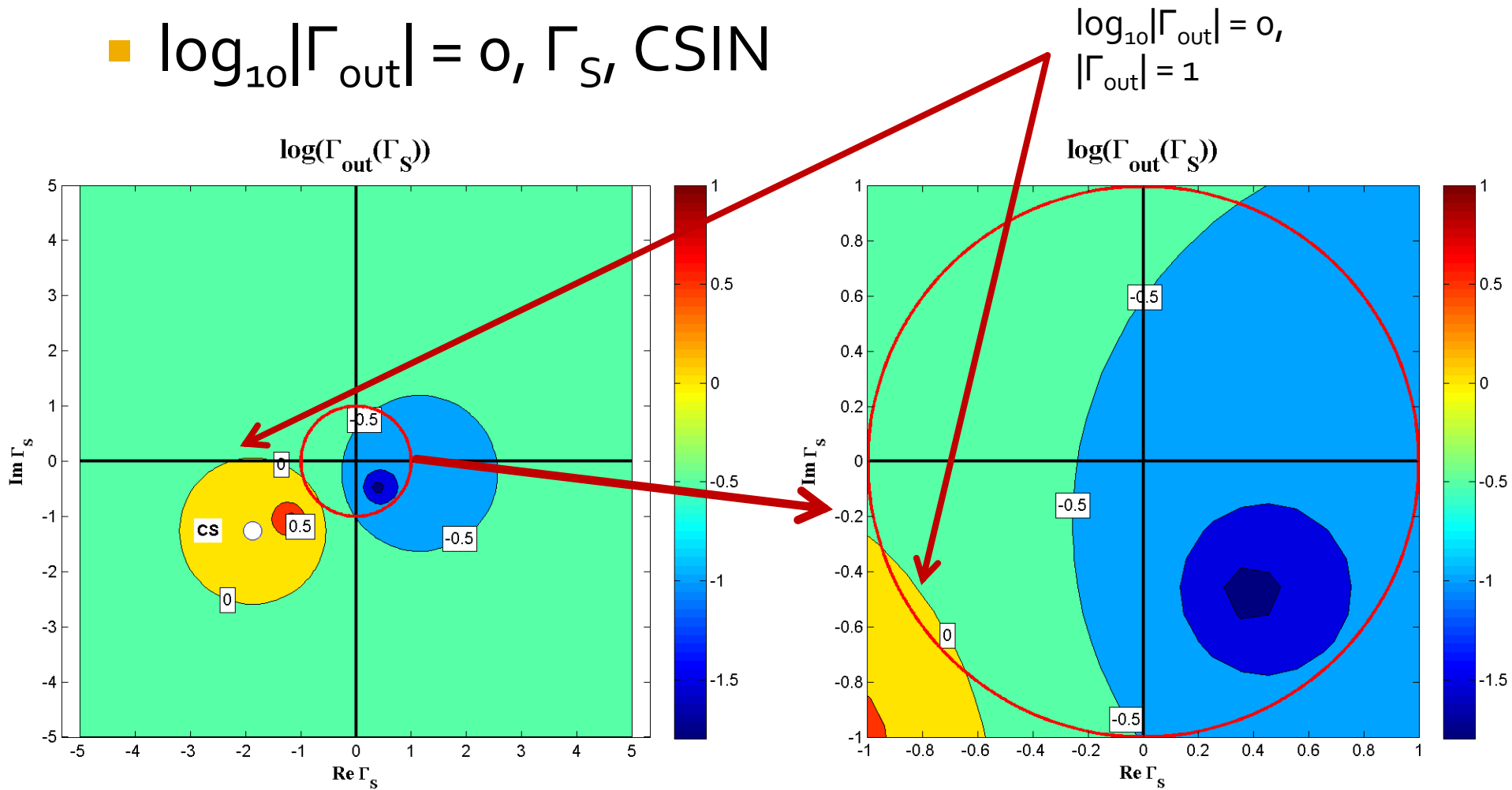
■ $\log_{10}|\Gamma_{in}| = 0, \Gamma_L, \text{CSOUT}$

$$\log_{10}|\Gamma_{in}| = 0, \\ |\Gamma_{in}| = 1$$



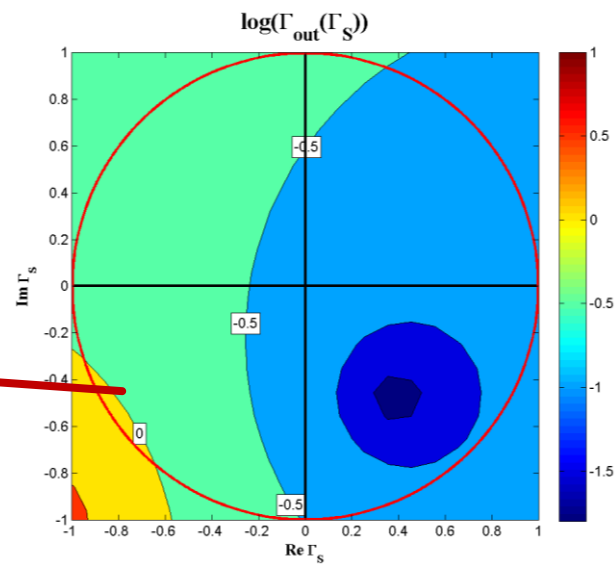
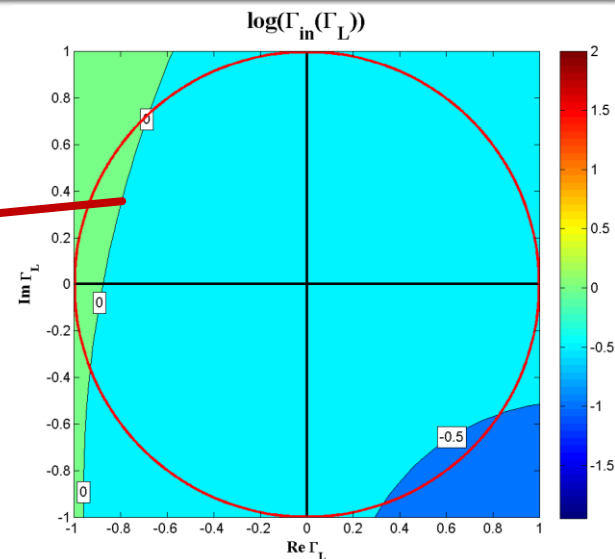
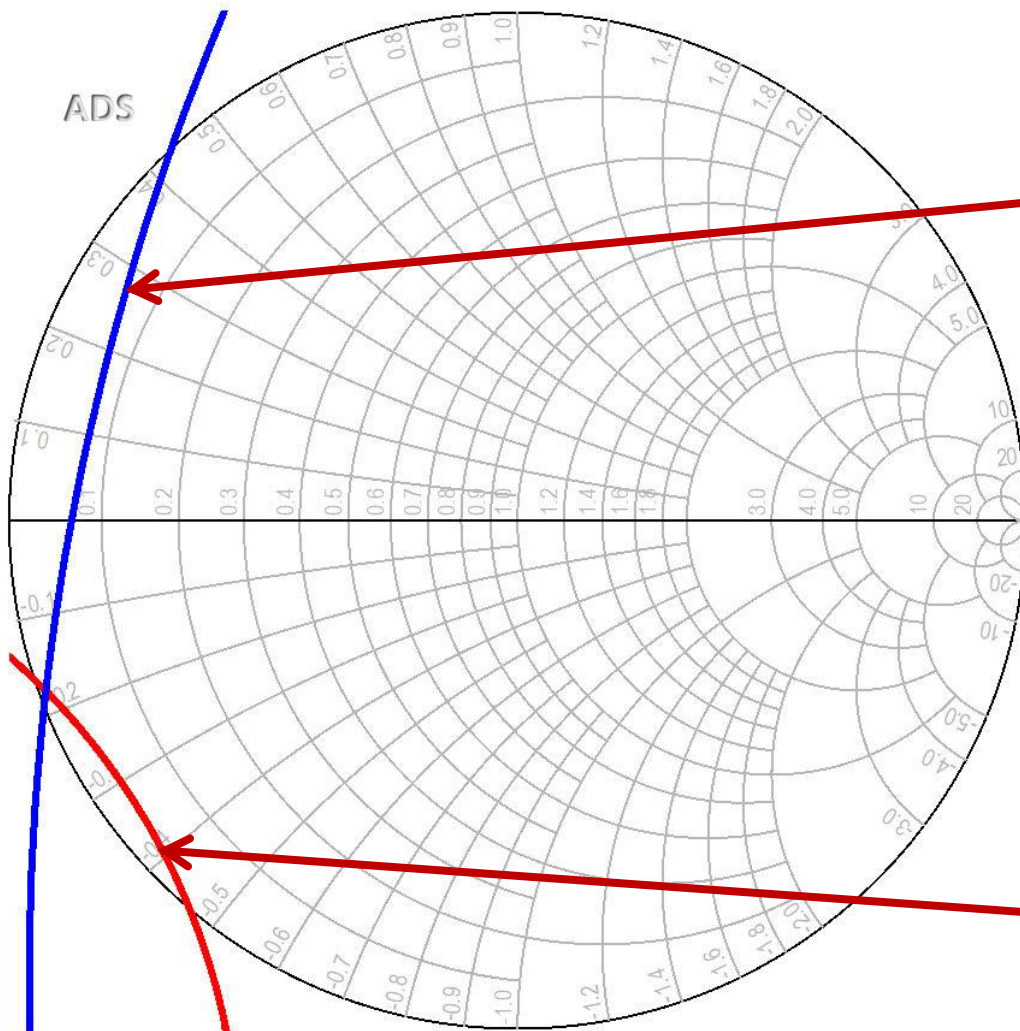
Contour lines of $\log_{10}|\Gamma_{out}|$

- $\log_{10}|\Gamma_{out}| = 0, \Gamma_S, \text{CSIN}$

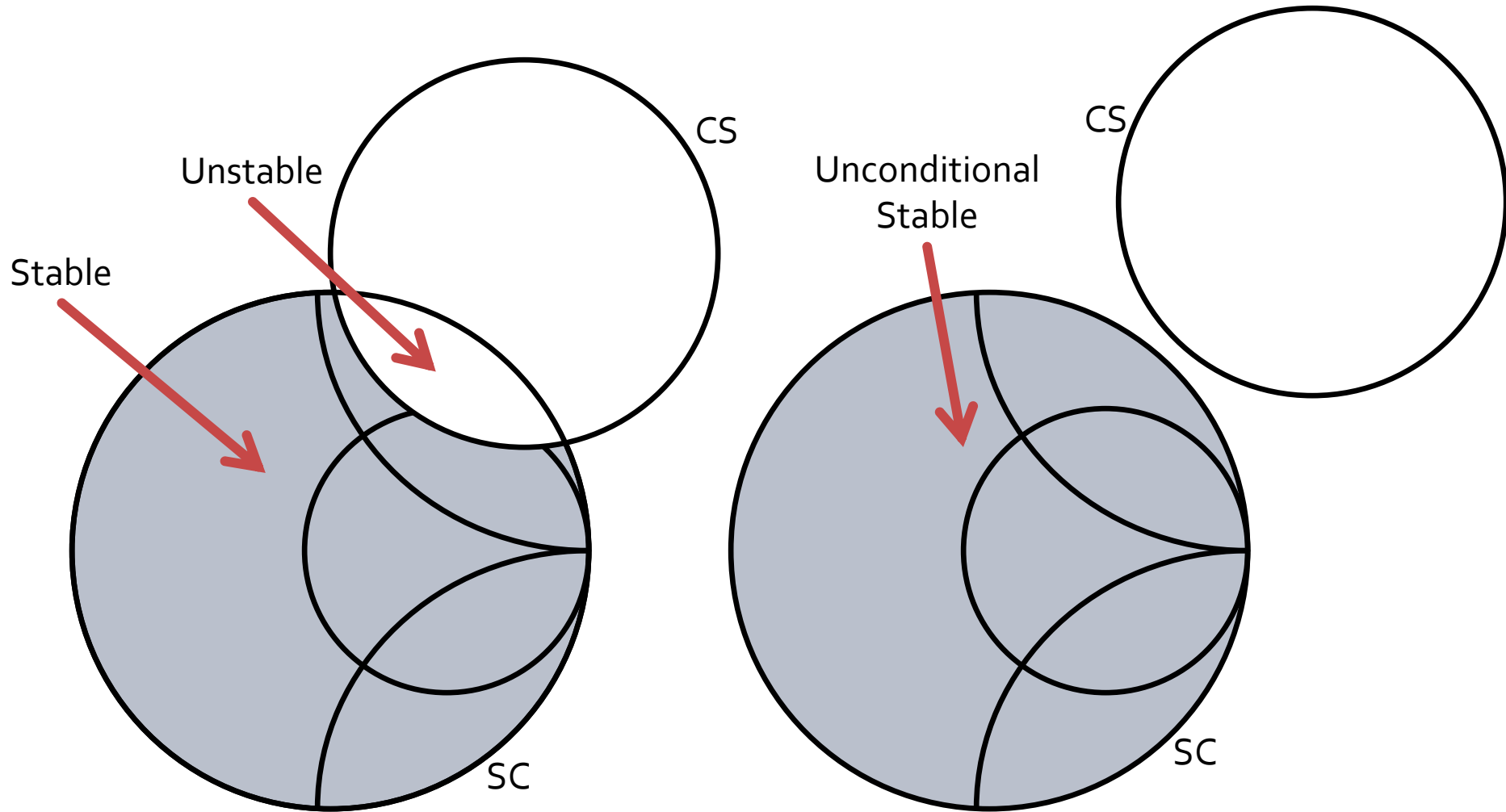


CSIN, CSOUT

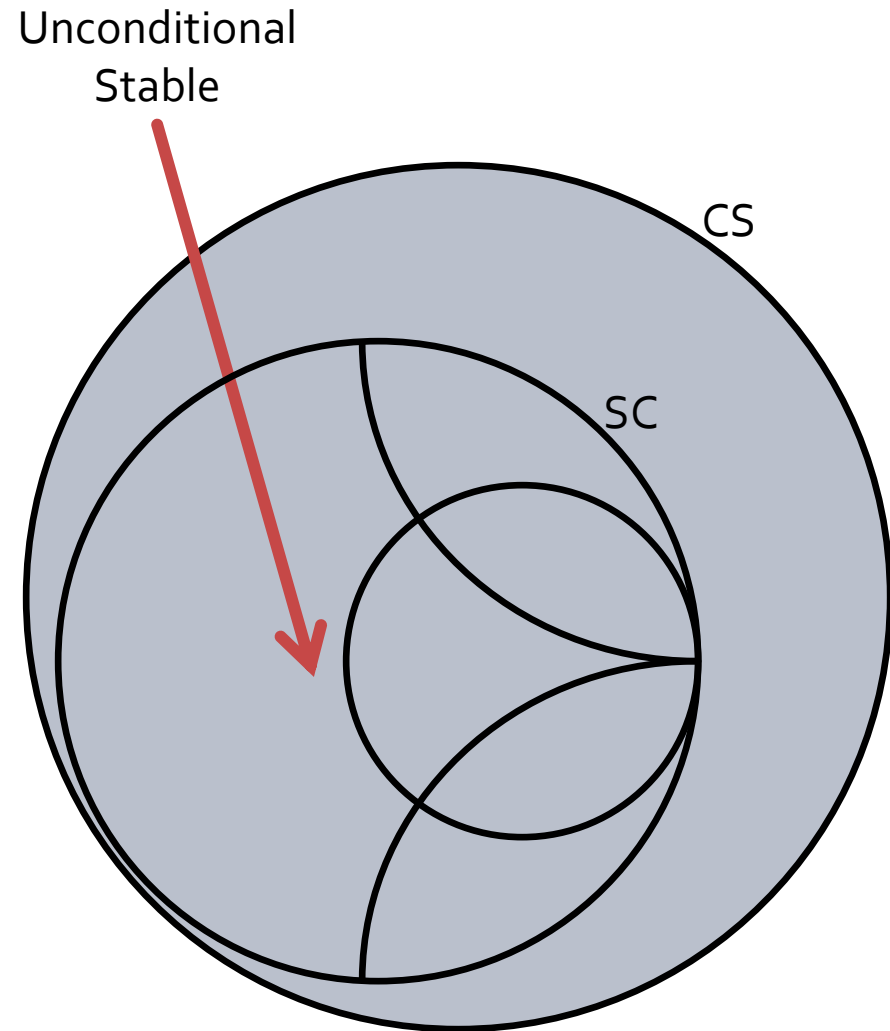
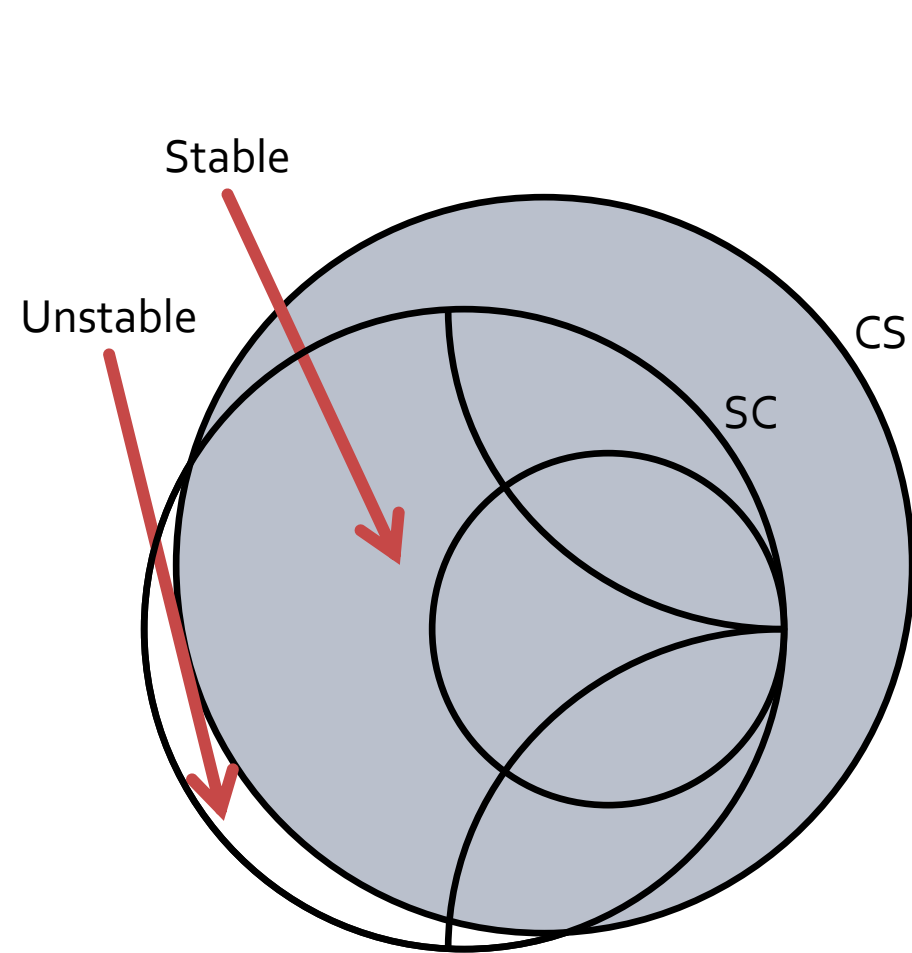
CSOUT
CSIN



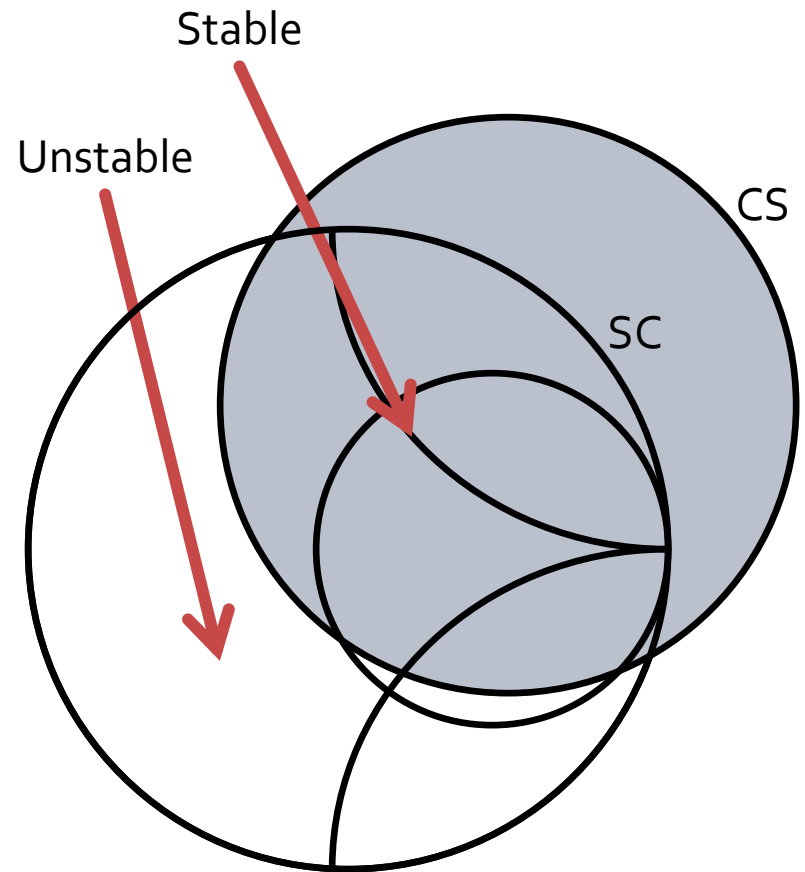
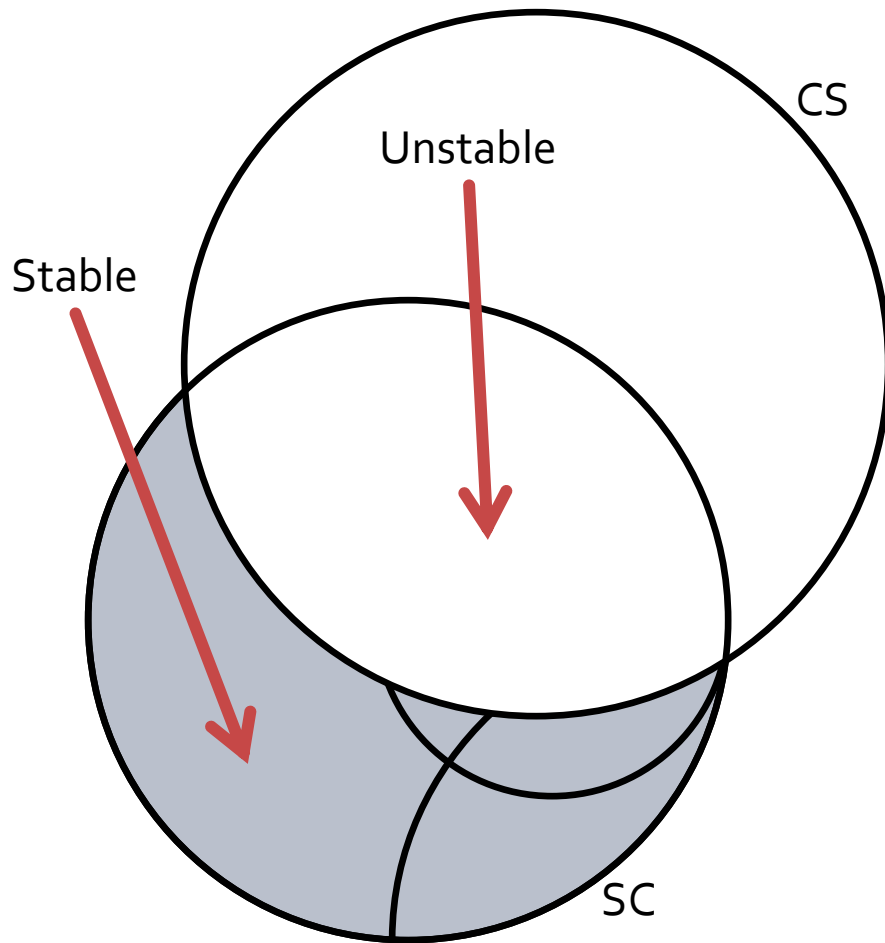
Several possible positioning



Several possible positioning



(Quite) Rare positioning



Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for **some** passive impedance of the load/source
 - passive impedance of the load/source \leftrightarrow interior of the Smith Chart (radius 1 circle in the complex plane)

Unconditional stability

- The two-port is unconditionally stable if either:
 - The stability circle is disjoint with the Smith Chart (exterior to the Chart) and the stable region is outside the circle
 - The stability circle encloses the entire Smith Chart and the stable region is inside the circle
- One mandatory condition for unconditional stability is $|S_{11}| < 1$ (CSOUT) or $|S_{22}| < 1$ (CSIN) – if in at least one point the two-port is not stable then it cannot be unconditionally stable

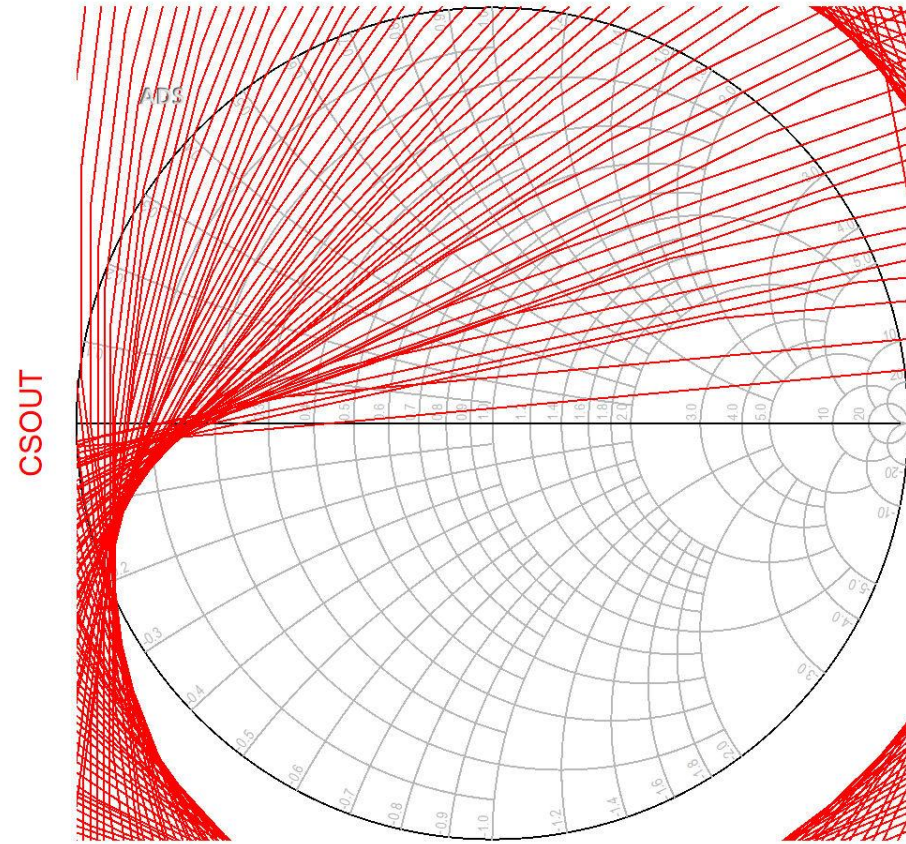
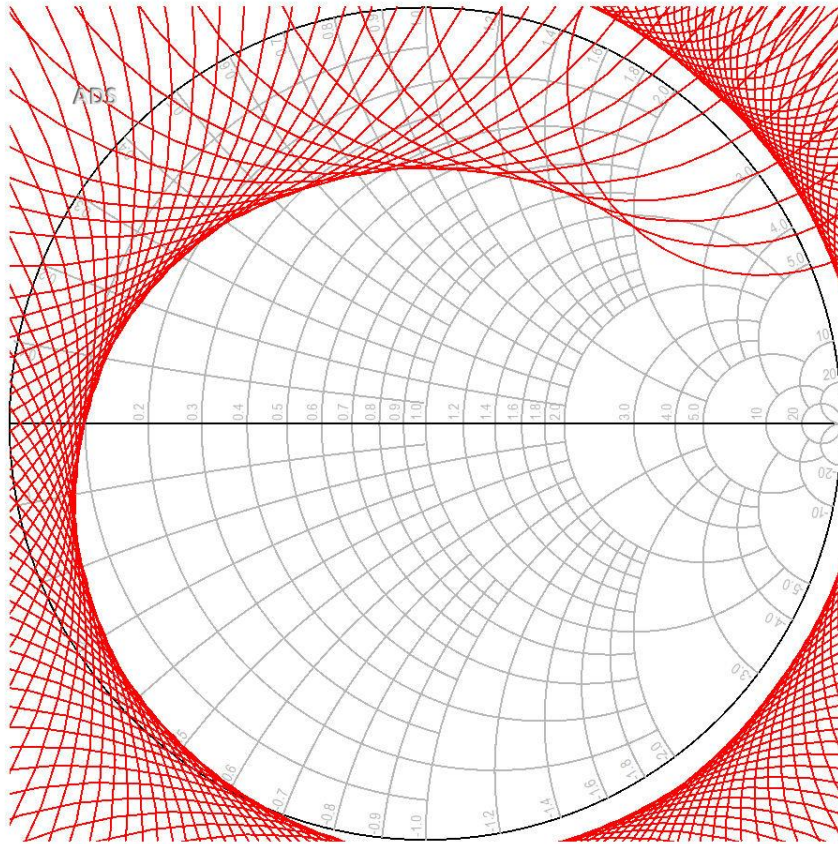
- Mathematically :

$$\left\{ \begin{array}{l} \left| |C_L| - R_L \right| > 1 \\ |S_{11}| < 1 \end{array} \right. \quad \left\{ \begin{array}{l} \left| |C_S| - R_S \right| > 1 \\ |S_{22}| < 1 \end{array} \right.$$

Tests for Unconditional Stability

- Useful for wide frequency range analysis
- It is not enough to check the stability only at the operating frequencies
 - we must obtain stable operation for chosen Γ_L and Γ_S at **any** frequency

Circles in wide frequency range



Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|}$$

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
 - $K > 1$
 - $|\Delta| < 1$
- together with the implicit conditions:
 - $|S_{11}| < 1$
 - $|S_{22}| < 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$$

$$|\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

μ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters, K and Δ

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta \cdot S_{11}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
 - $\mu > 1$
- together with the implicit conditions:
 - $|S_{11}| < 1$
 - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ imply greater stability
 - μ is the distance from the center of the Smith Chart to the closest output stability circle

μ' Criterion

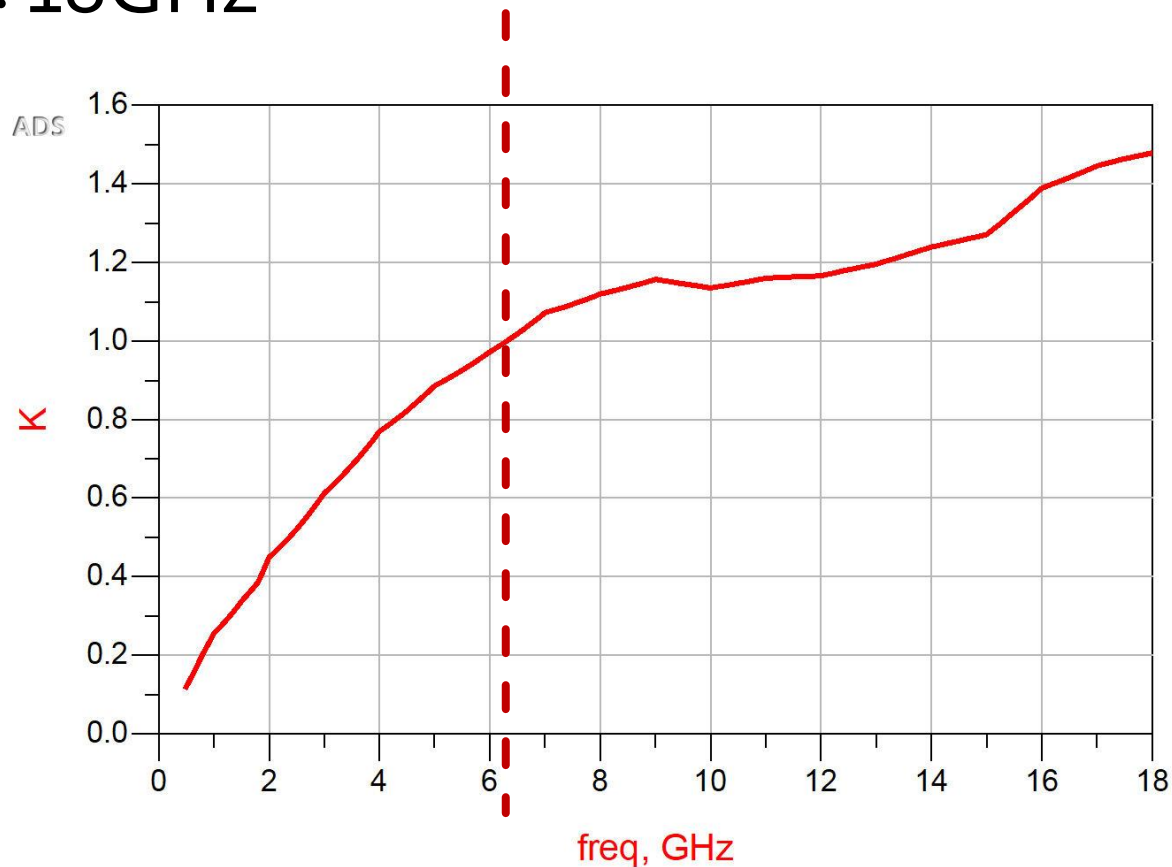
- Dual parameter to μ , determined in relation to the input stability circles

$$\mu' = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta \cdot S_{22}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
 - $\mu' > 1$
- together with the implicit conditions:
 - $|S_{11}| < 1$
 - $|S_{22}| < 1$
- In addition, it can be said that larger values of μ' imply greater stability
 - μ' is the distance from the center of the Smith Chart to the closest input stability circle

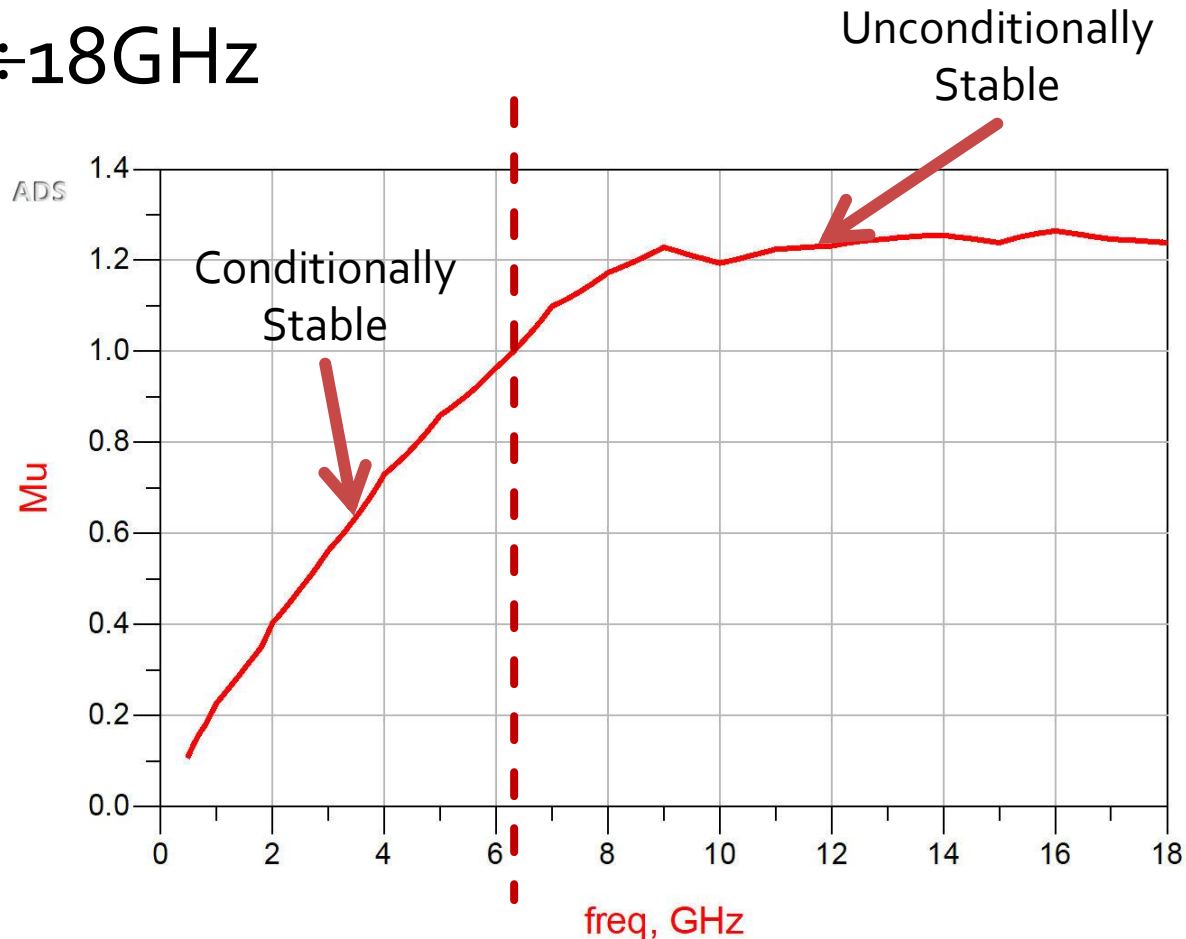
Rollet's condition

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @0.5÷18GHz



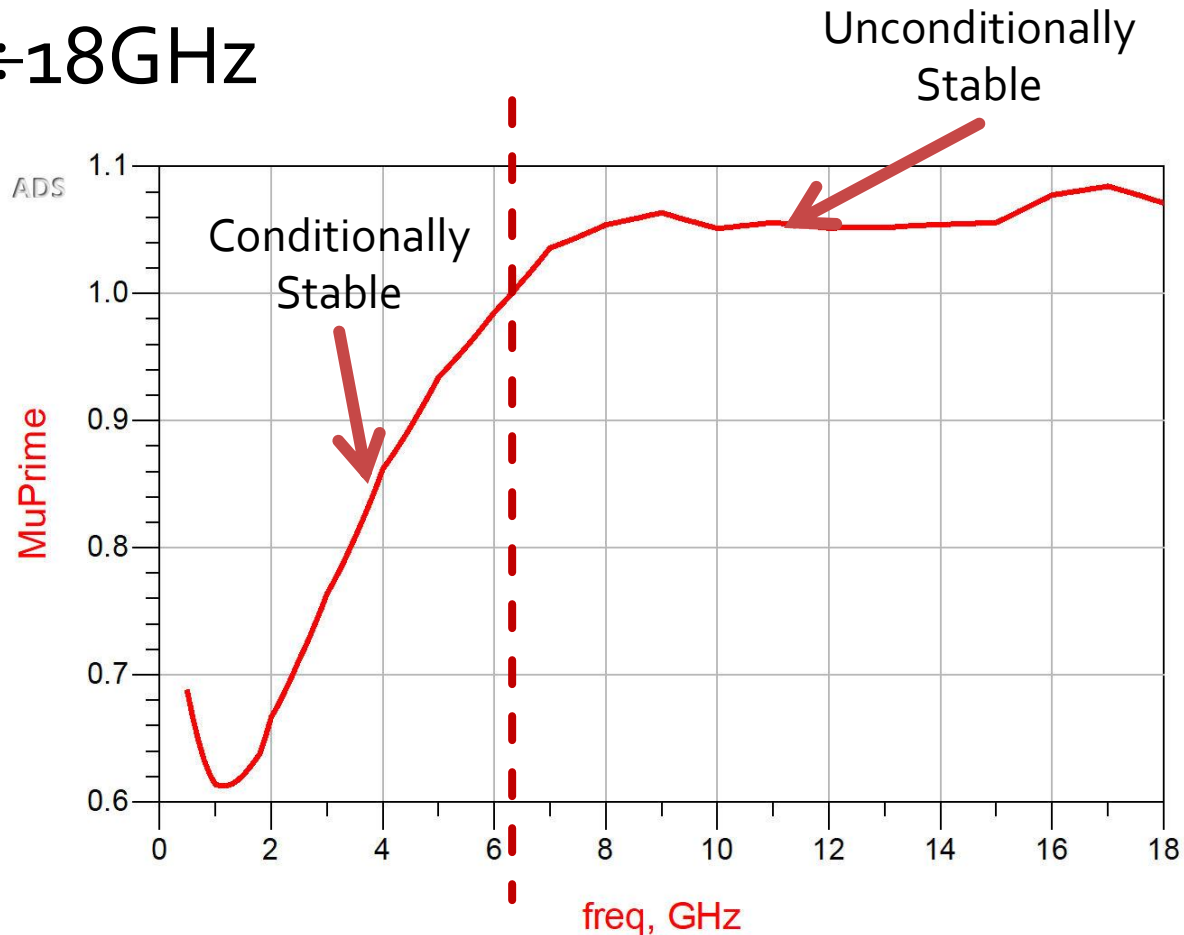
μ Criterion

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @0.5÷18GHz



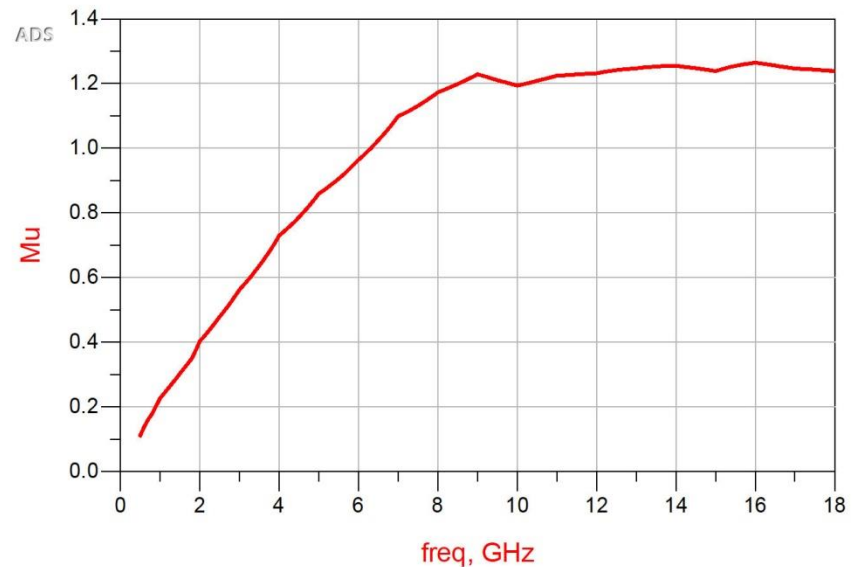
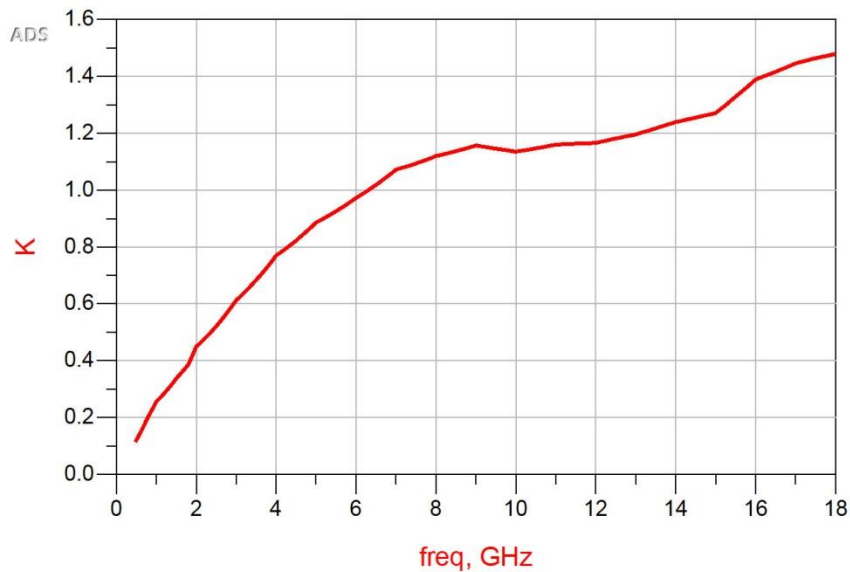
μ' Criterion

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @0.5÷18GHz



Stability

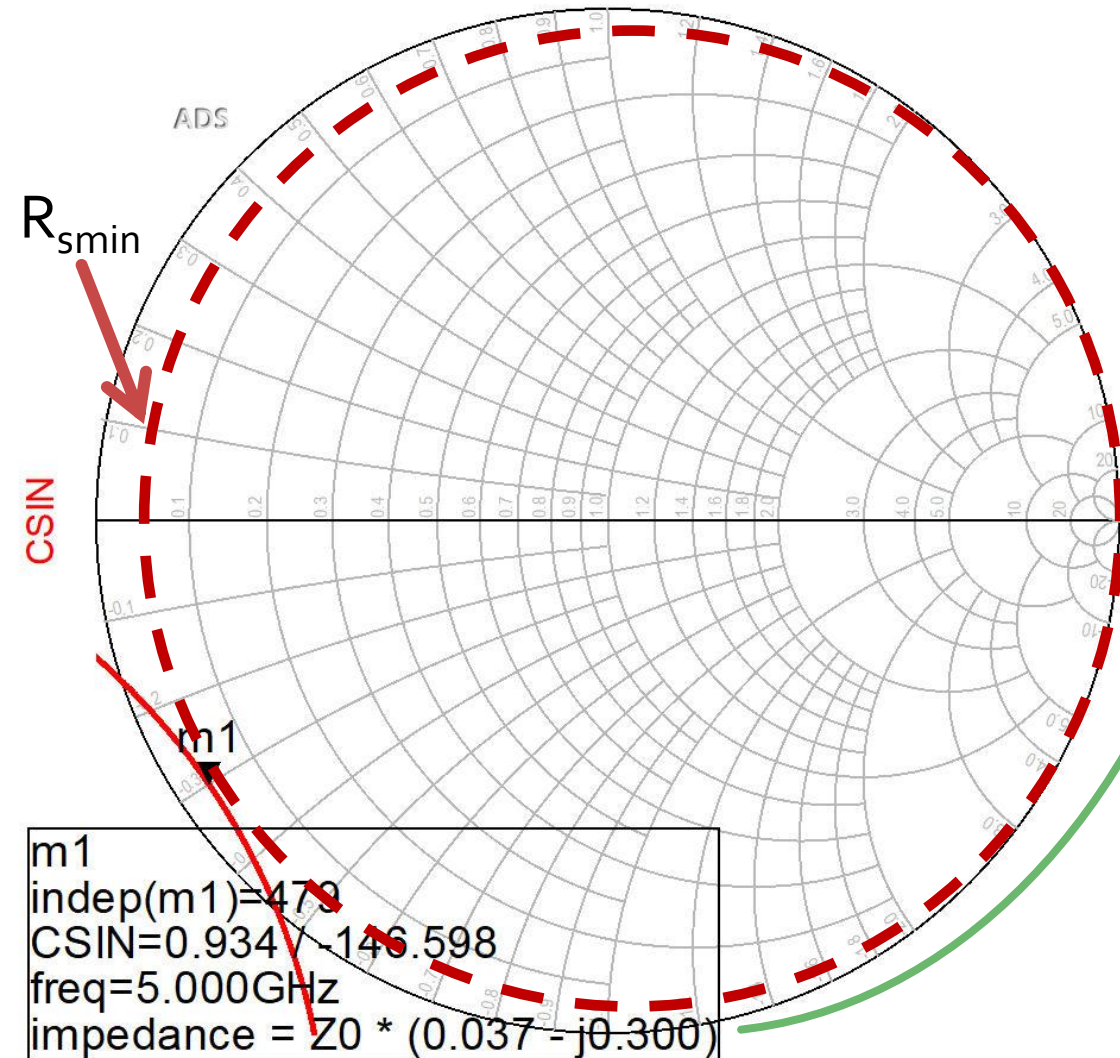
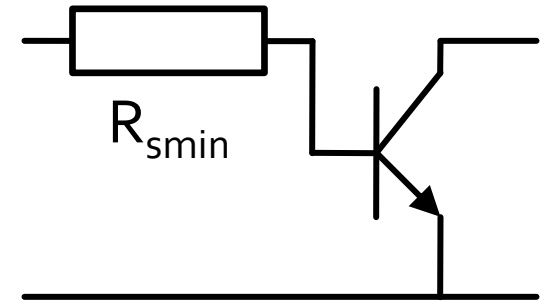
- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @0.5÷18GHz
- unconditionally stable for $f > 6.31GHz$



Stabilization of two-port

- Unconditional stability in a wide frequency range has some important advantages
 - Ex: We can use ATF 34143 to design a (conditionally) stable amplifier at 5GHz, but this design is useless if the amplifier oscillates at 500MHz ($\mu \approx 0.1$)
- **The minimal requirement** when working with conditionally stable devices is to **check stability** at several frequencies over the operating bandwidth and outside the bandwidth
- Unconditional stability can be forced by inserting series/shunt resistors at two-port's input/output (with loss of gain!)

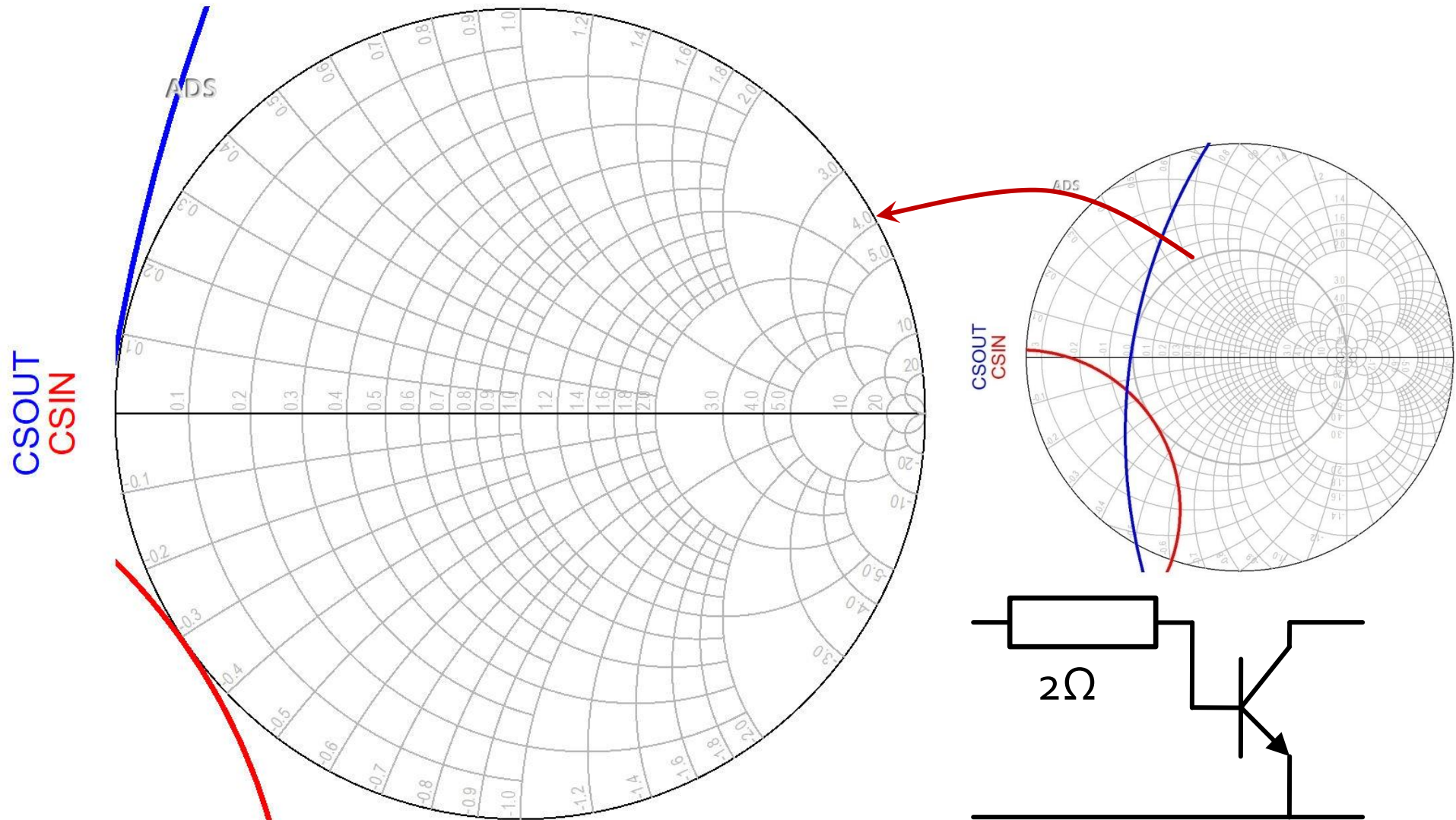
Input series resistor



$$z = 0.037 - j \cdot 0.3$$

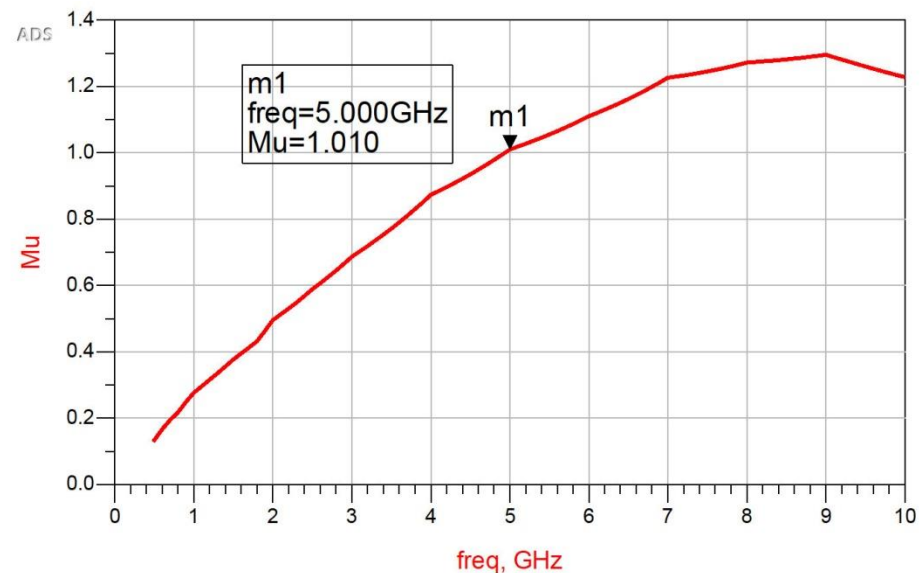
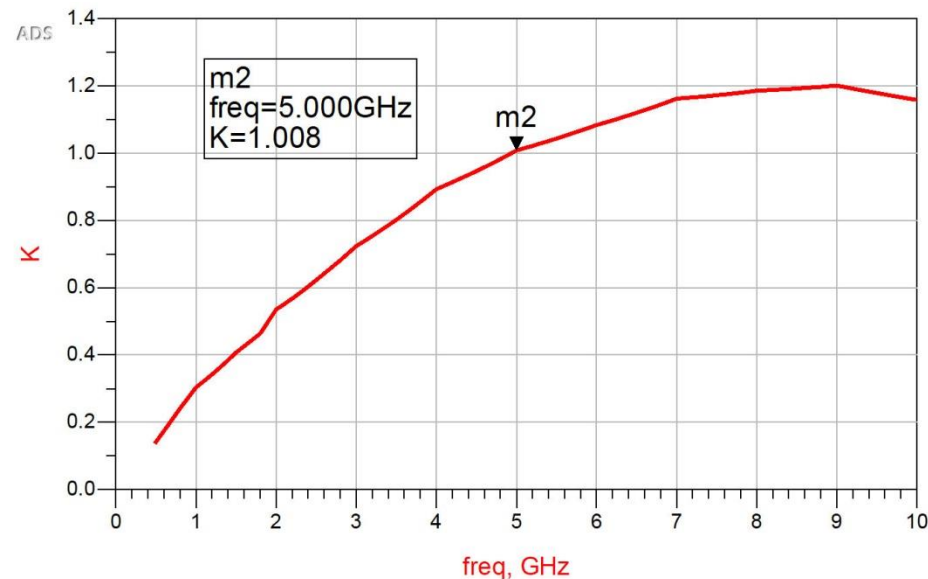
$$R_{smin} = 0.037 \cdot 50\Omega = 1.85\Omega$$

ADS, $R_s = 2\Omega$

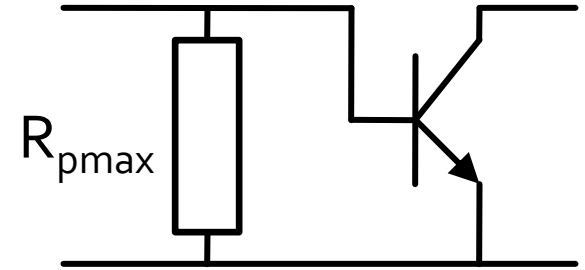
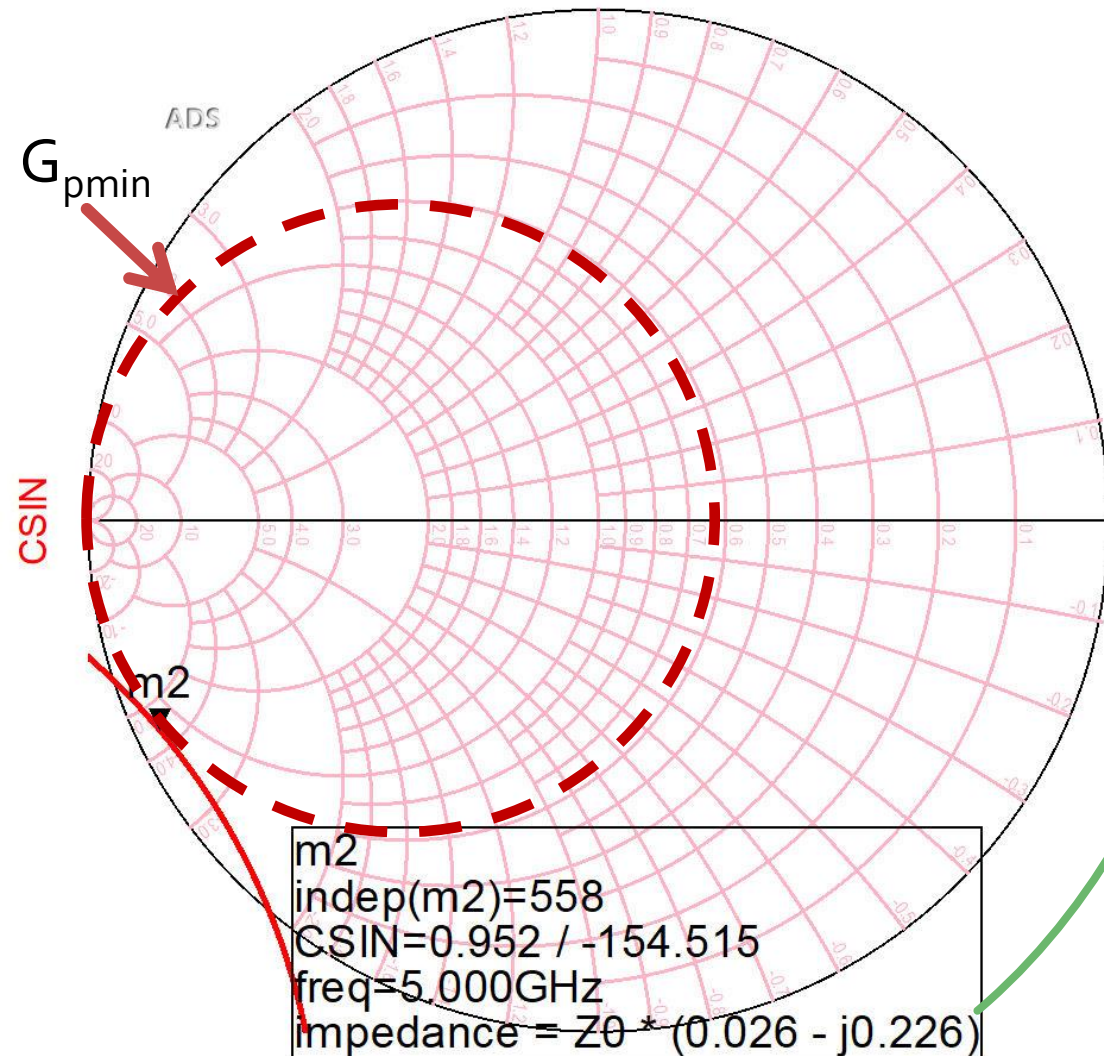


Input series resistor

- $R_s = 2\Omega$
- $K = 1.008$, $MAG = 13.694\text{dB}$ @ 5GHz
 - no stabilization, $K = 0.886$, $MAG = 14.248\text{dB}$ @ 5GHz



Input shunt resistor



$$R_{pmax} = \frac{1}{G_{pmin}}$$

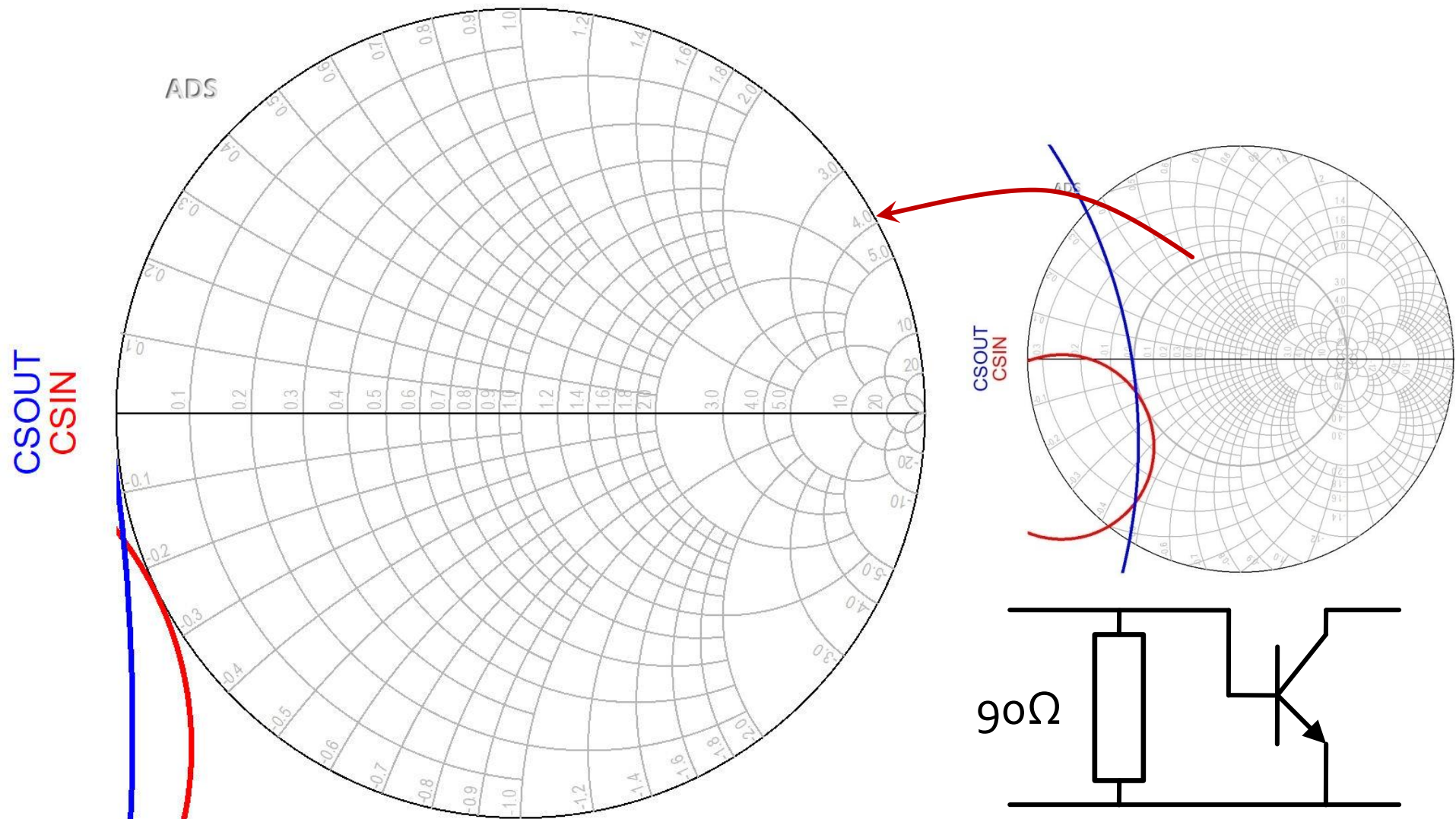
$$z = 0.026 - j \cdot 0.226$$

$$y = \frac{1}{z} = \frac{1}{0.026 - j \cdot 0.226}$$

$$y = 0.502 + j \cdot 4.367$$

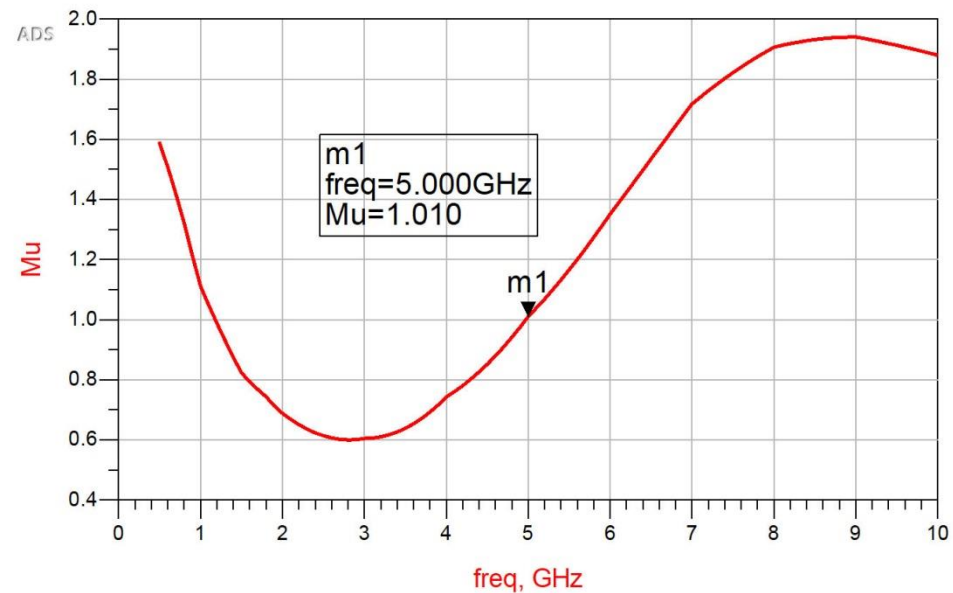
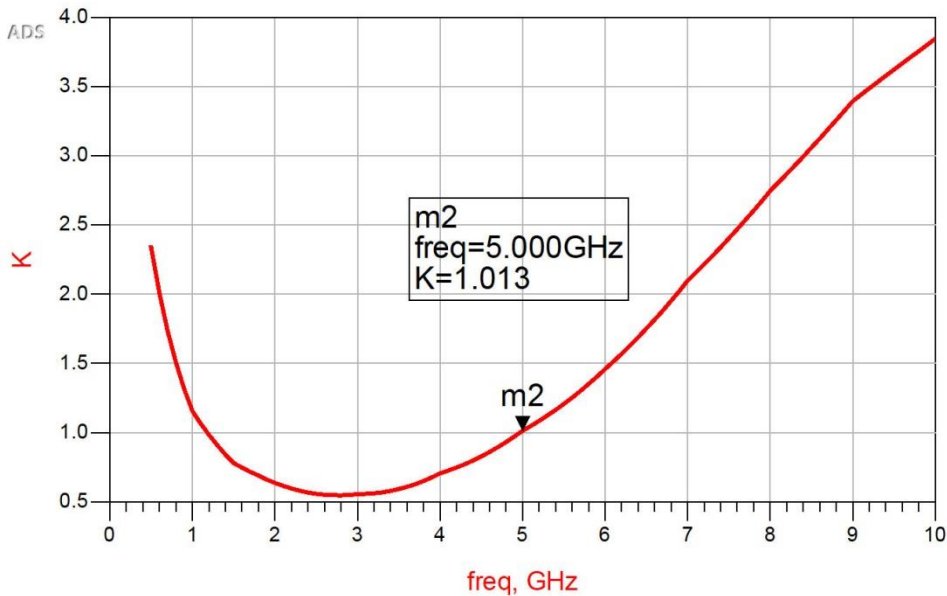
$$R_{pmax} = \frac{50\Omega}{0.502} = 99.6\Omega$$

ADS, $R_p = 90\Omega$



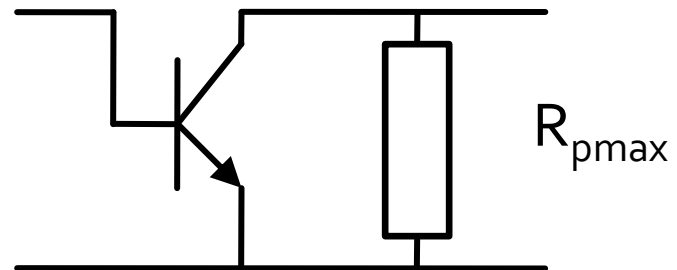
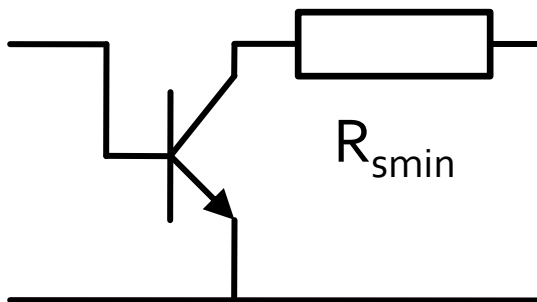
Input shunt resistor

- $R_p = 90\Omega$
- $K = 1.013$, $MAG = 13.561\text{dB}$ @ 5GHz
 - no stabilization, $K = 0.886$, $MAG = 14.248\text{dB}$ @ 5GHz



Output series/shunt resistor

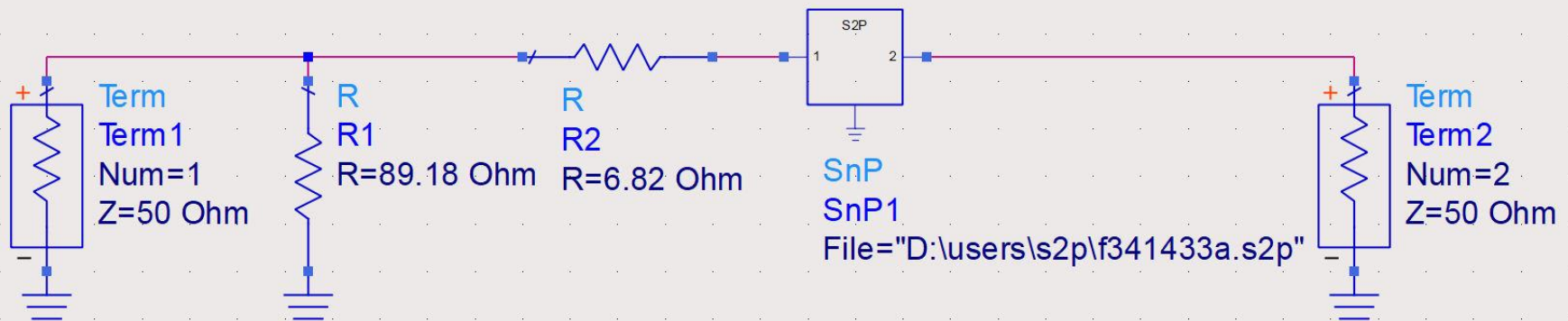
- The procedure can be applied similarly at the output (finding g/r circles tangent to CSOUT)
- From previous examples, resistive loading at the input has a positive effect over output stability and vice versa (resistive loading at the output, effect over input stability)



Stabilization of two-port

- Negative effect over the power gain
 - we must check MAG/MSG while designing resistive loading
- Negative effect over the noise (debated next)
- We can choose one of the 4 possibilities or a combination which offers better results (depending on transistor, application etc.)
- We can use frequency selective loading
 - Ex: RL, RC circuits which sacrifice performance only when needed to improve stability and have no effect at frequencies where the device is already stable
- It might be possible (and should be checked) that stability is improved as an effect of parasitic elements of biasing circuits (bypass capacitors and RF chokes)

Stabilization of two-port



S-PARAMETERS

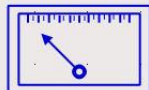
S_Param

SP1

Start=0.5 GHz

Stop=10.0 GHz

Step=0.1 GHz

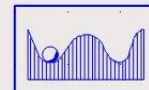


MaxGain

MaxGain

MAG

MAG=max_gain(S)

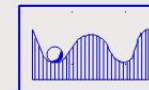


Mu

Mu

Mu1

Mu=mu(S)



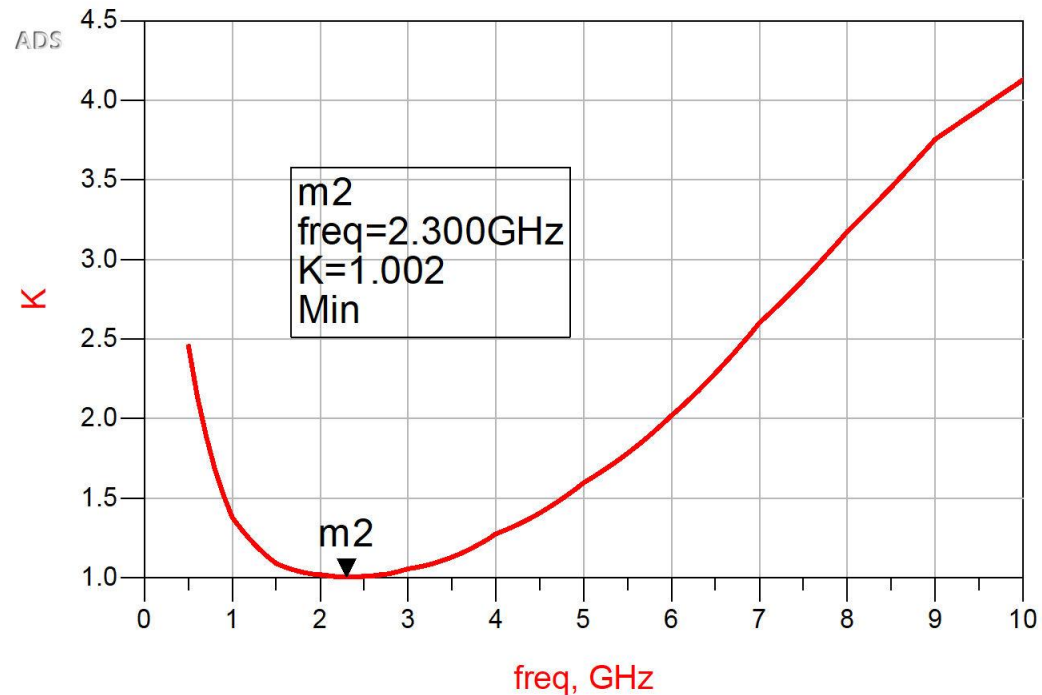
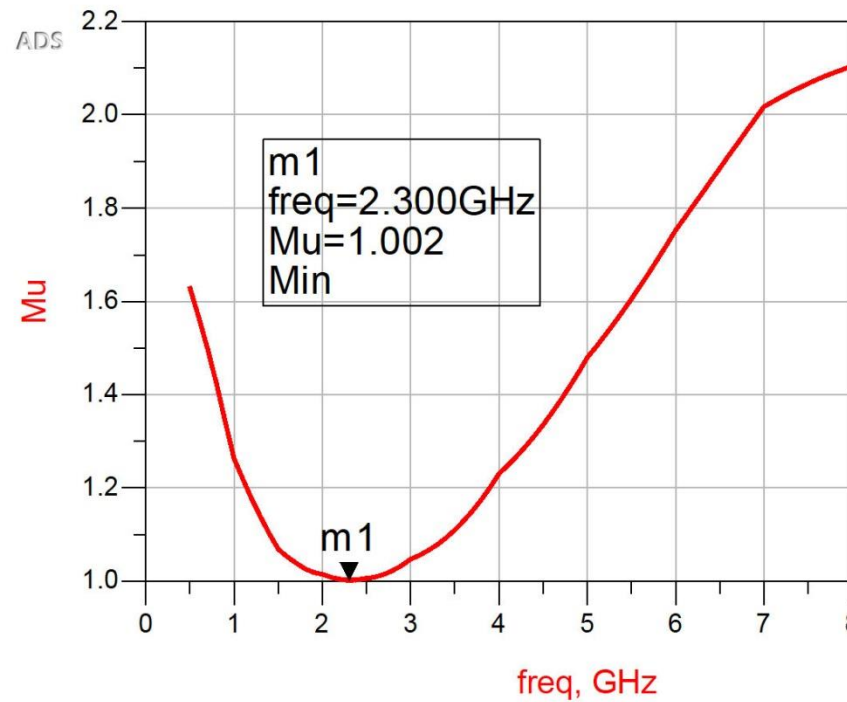
StabFact

StabFact

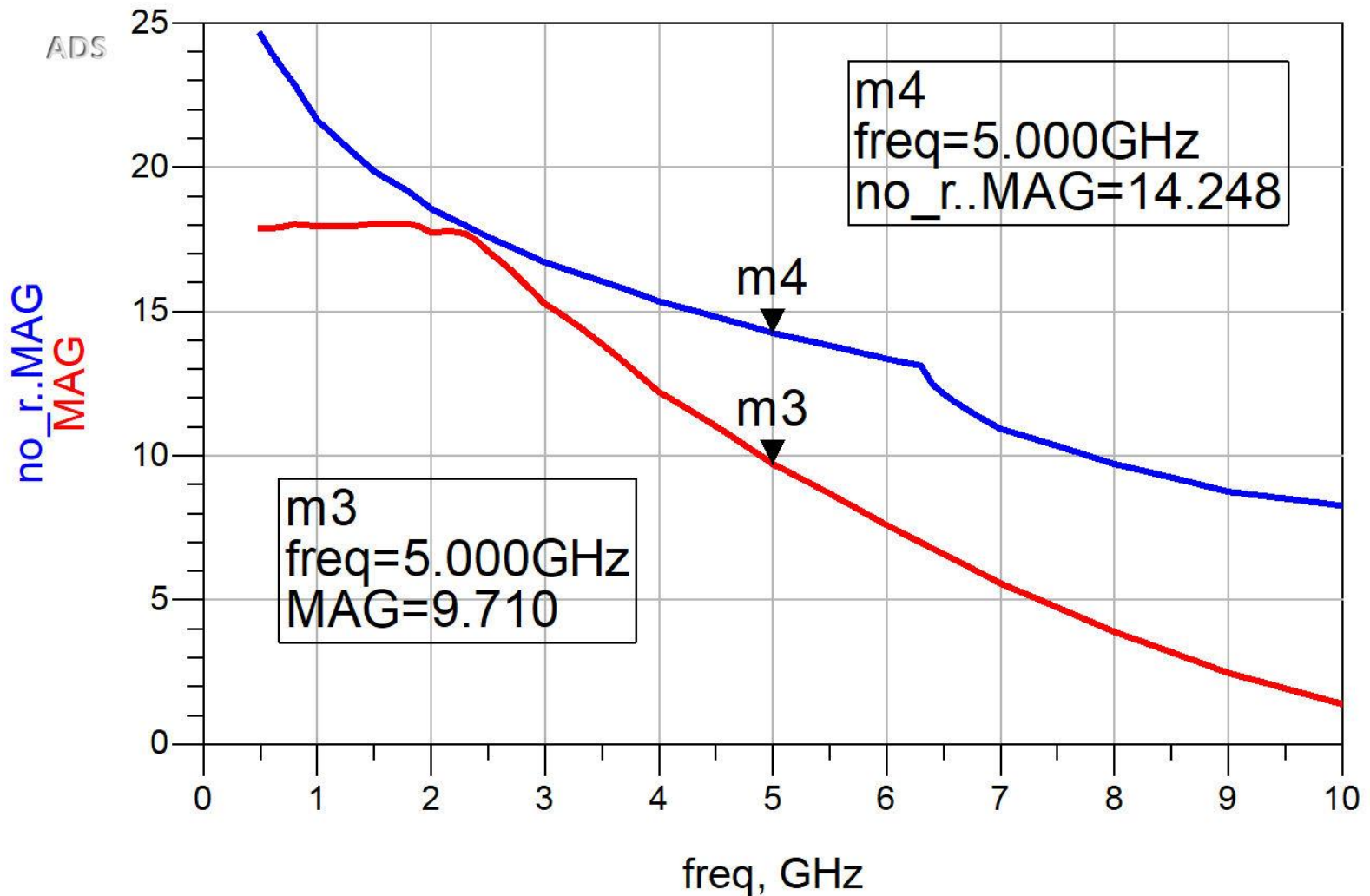
K

K=stab_fact(S)

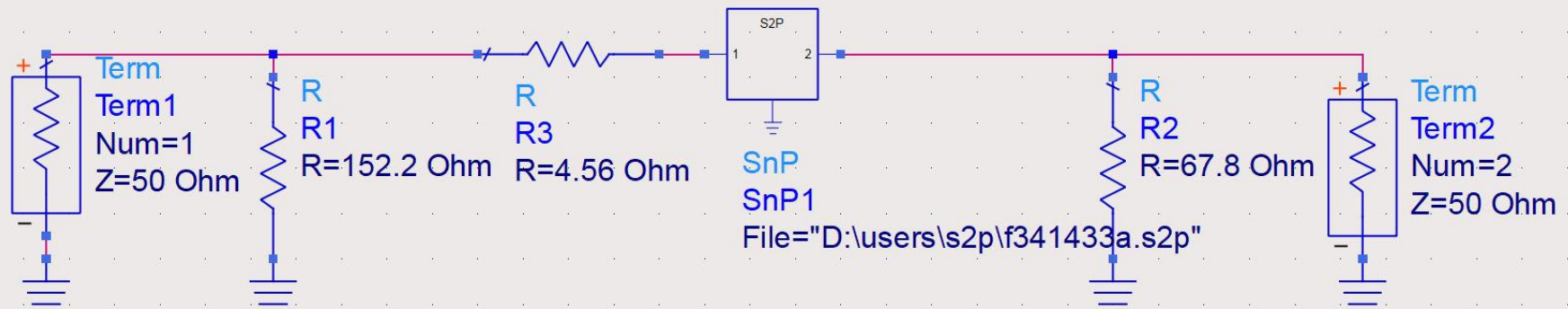
Stabilization of two-port




Stabilization of two-port

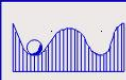


Stabilization of two-port




 **S-PARAMETERS**

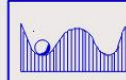
S_Param
SP1
Start=0.5 GHz
Stop=10.0 GHz
Step=0.1 GHz

 **StabFact**

StabFact
K
K=stab_fact(S)

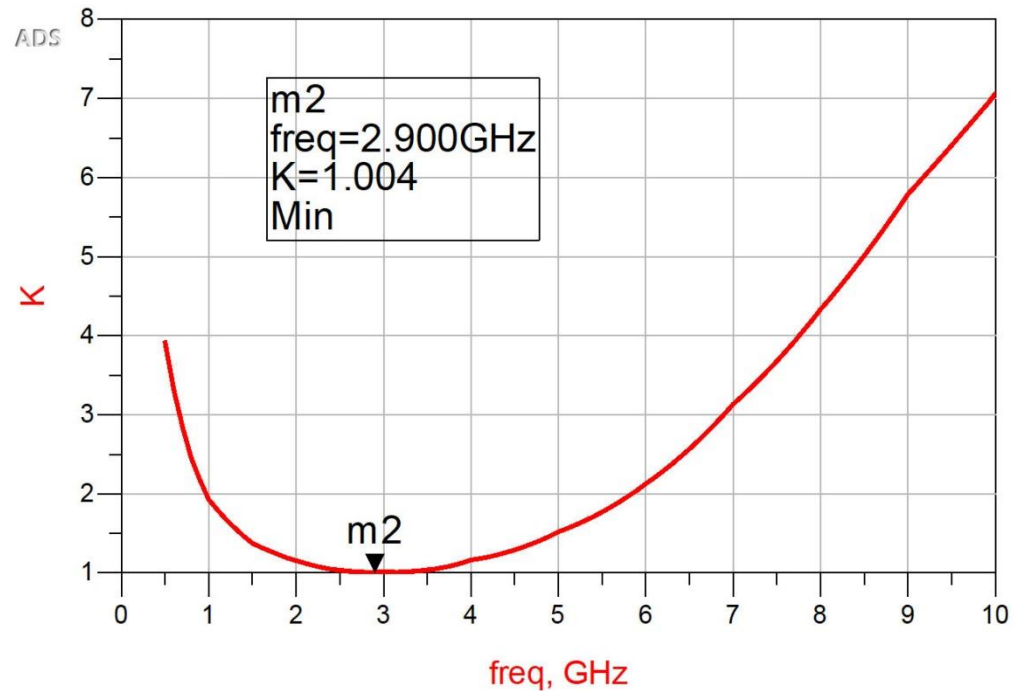
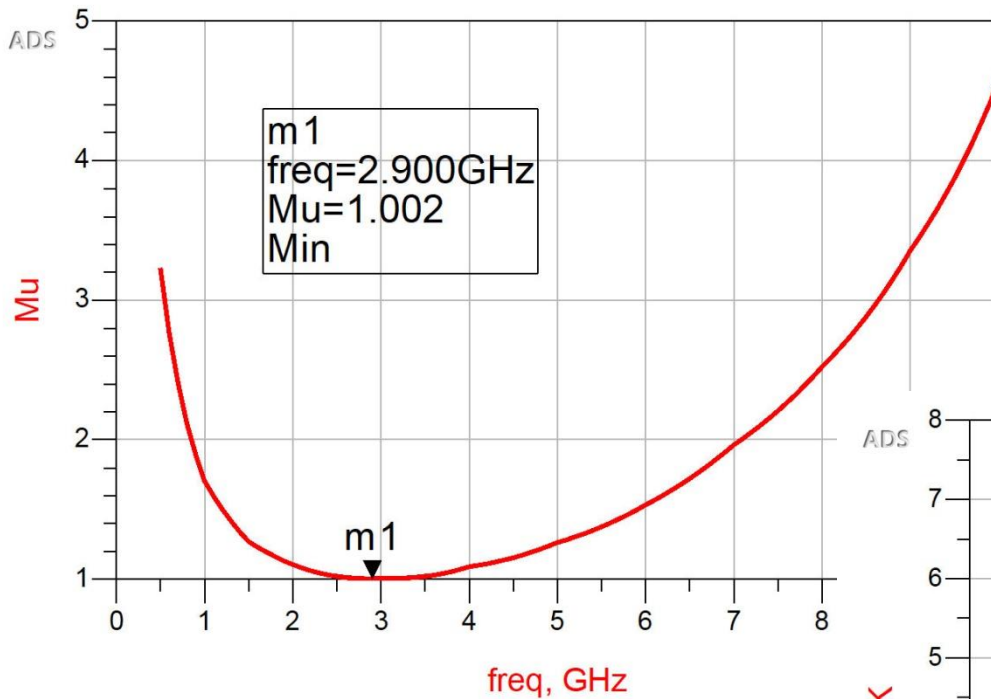
 **MaxGain**

MaxGain
MAG
MAG=max_gain(S)

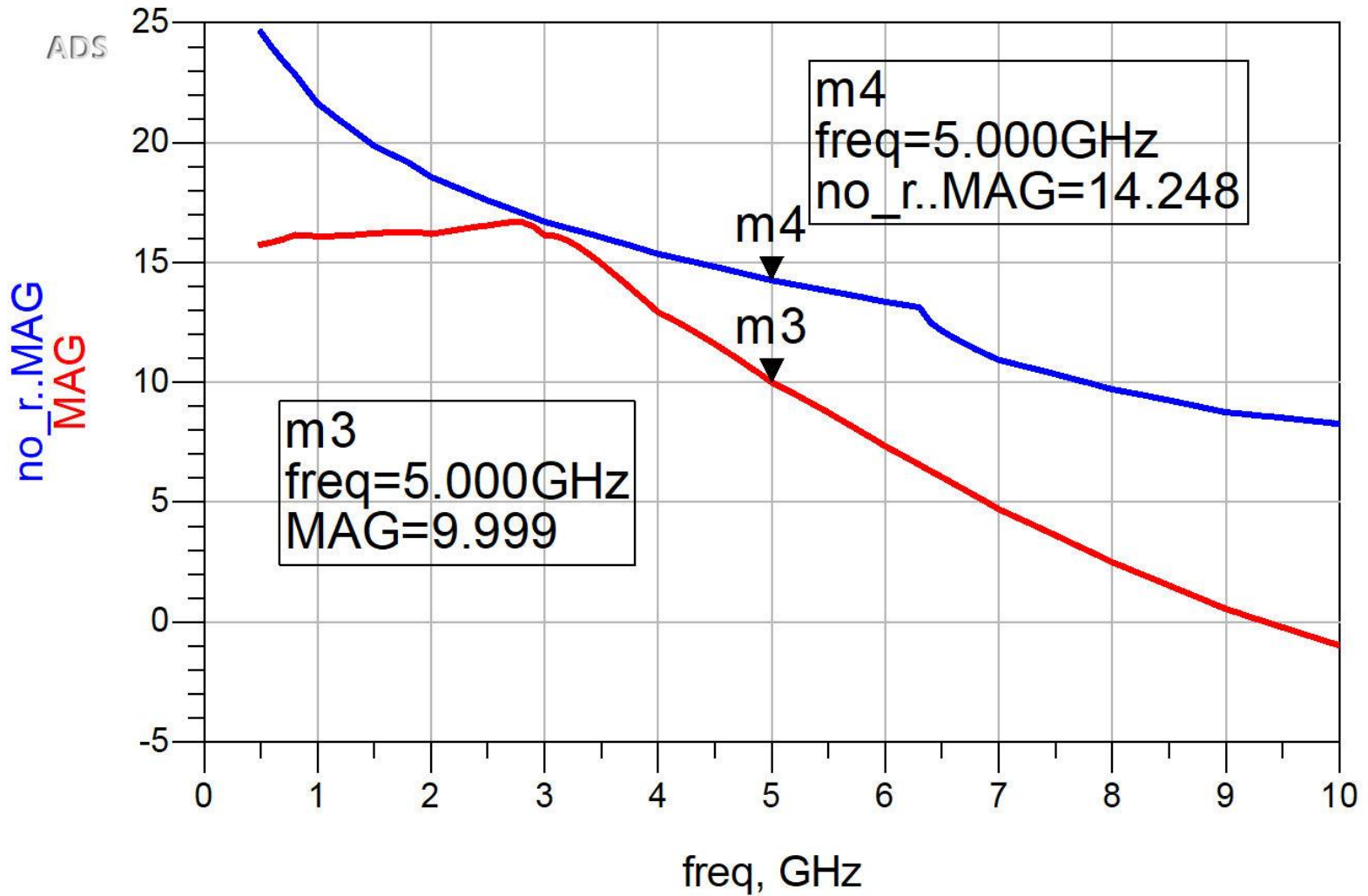
 **Mu**

Mu
Mu1
Mu=mu(S)

Stabilization of two-port



Stabilization of two-port



Contact

- Microwave and Optoelectronics Laboratory
- <http://rf-opto.etti.tuiasi.ro>
- rdamian@etti.tuiasi.ro